Modeling and Analysis of Hybrid Systems

Introduction

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Informatik 2 - Theory of Hybrid Systems
RWTH Aachen University

SS 2011

Organizational

Lecture:

- Tuesday 13:15-14:15 in 5056
- Friday 13:15-14:30 in 5056

Exercise:

- Tuesday 14:15-15:00 in 5056

“Hybrid”

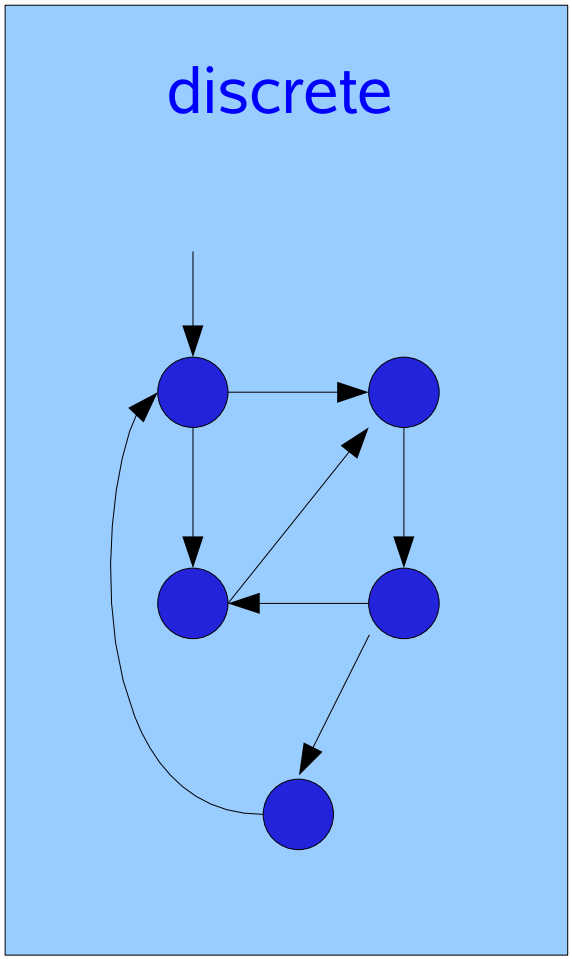
Wikipedia:

> “A hybrid is the combination of two or more different things, aimed at achieving a particular objective or goal.”

A hybrid rose

A hybrid car

Hybrid in computer science



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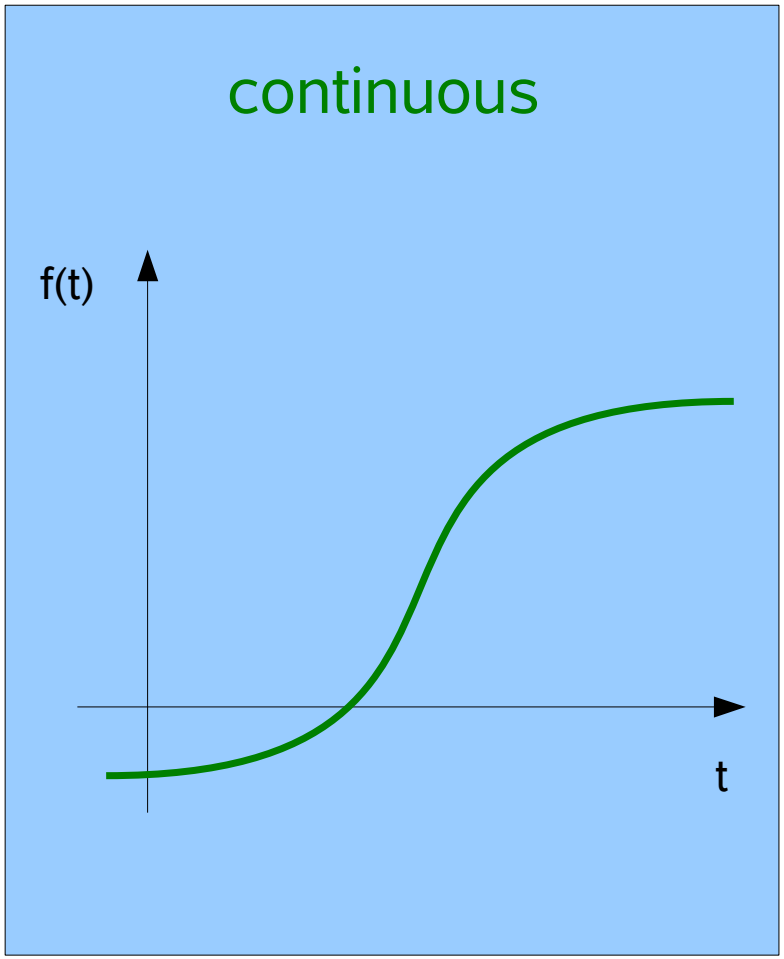
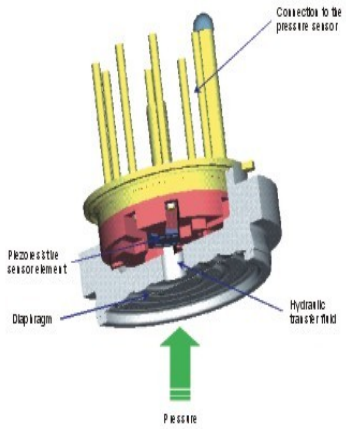
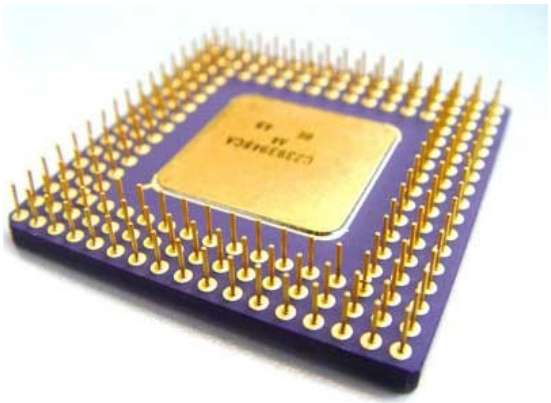


The discrete part



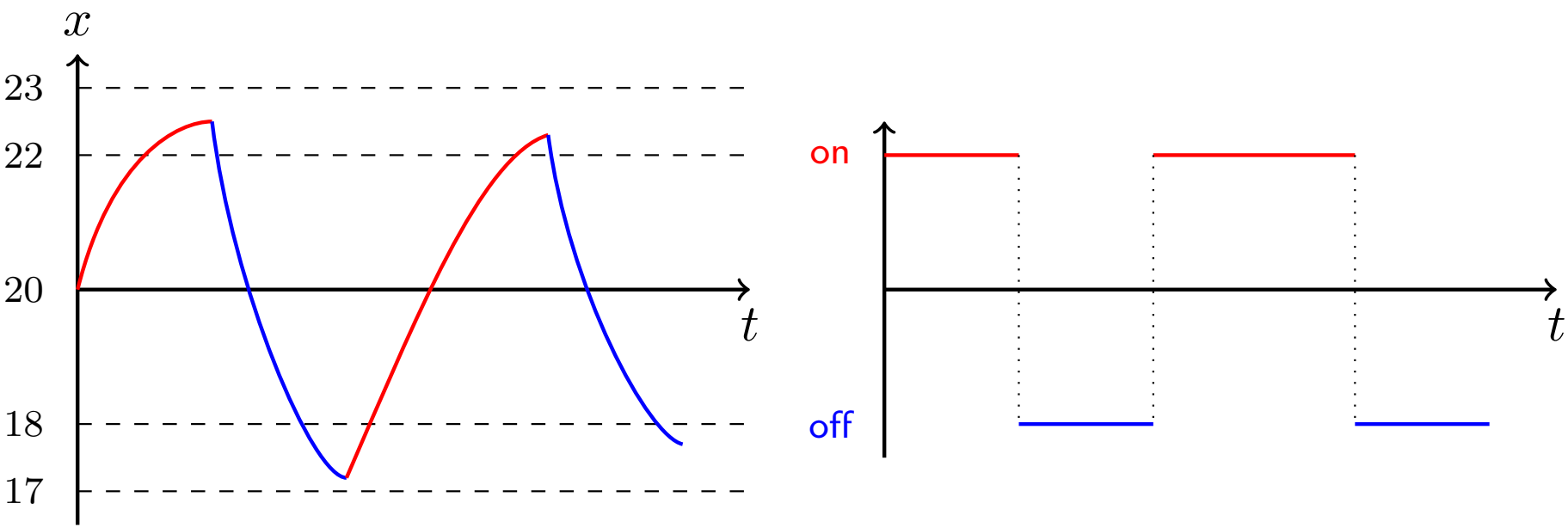


Combined with the continuous part

Example: Thermostat

- Temperature x is controlled by switching a heater on and off
- x is regulated by a thermostat:
  - $17° \leq x \leq 18° \rightsquigarrow$ "heater on"
  - $22° \leq x \leq 23° \rightsquigarrow$ "heater off"

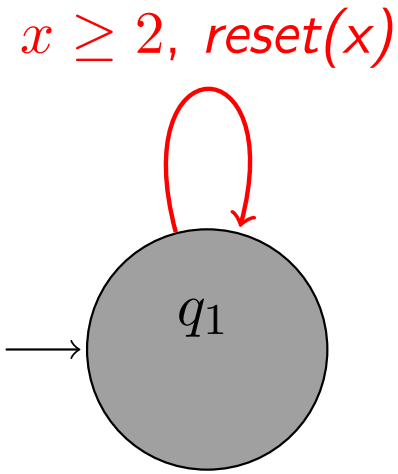
Continuous: temperature
Discrete: switching

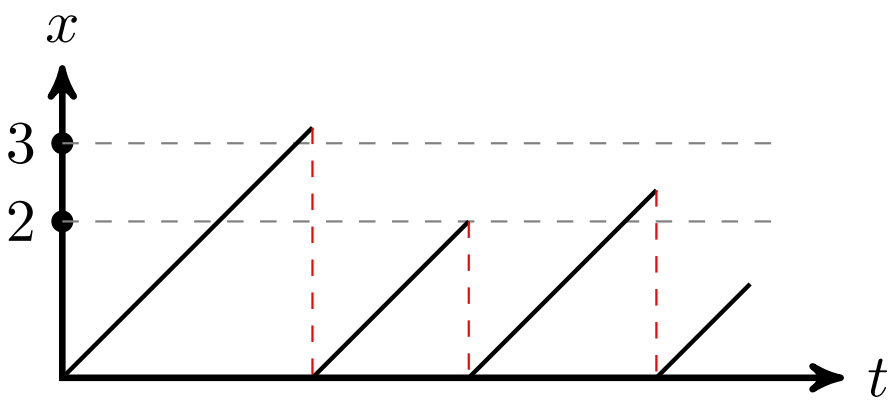


Modeling

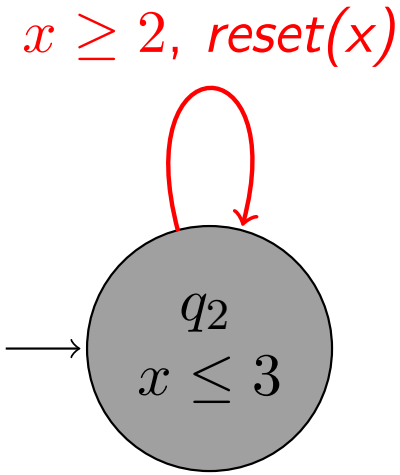
- To be able to apply formal (mathematical) methods to a real system, we need a formal model of it
- A model never exactly corresponds to the modeled real system
- Abstract away unnecessary details
- What you probably already know: Kripke structures (state transition systems)
- What you probably also know: Transition systems
- What you perhaps know: Timed automata

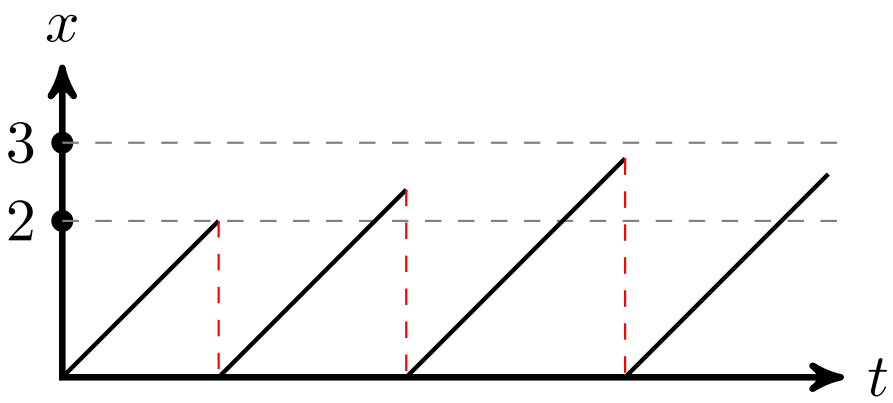
Example: Timed automaton





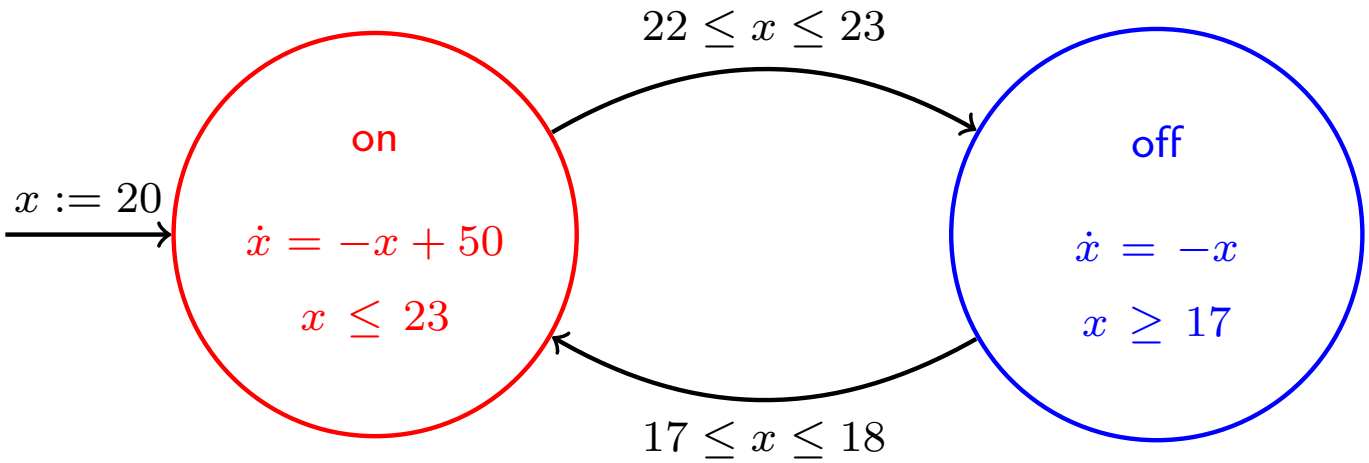
Example: Timed automaton

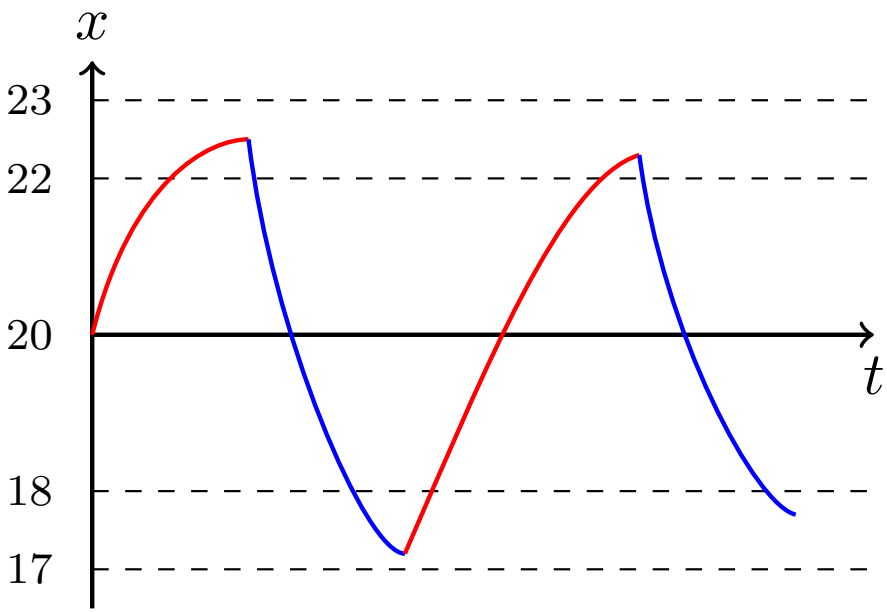


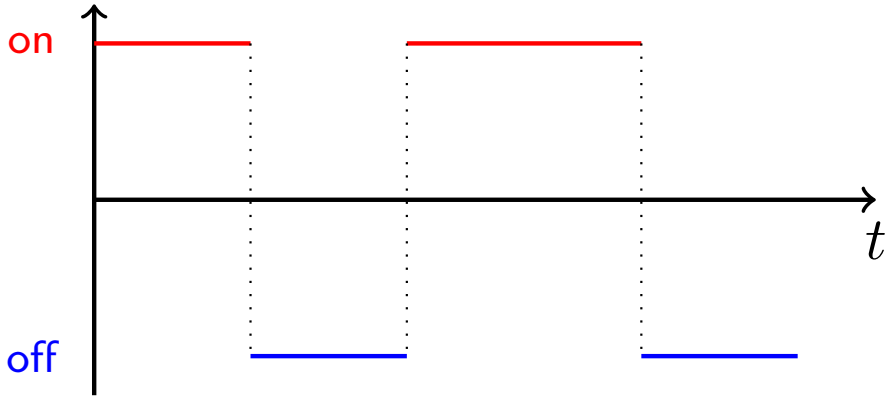


Modeling general hybrid systems: Hybrid automata

Let's take again the thermostat as an example.







Logic

- We want to specify how a hybrid system is expected to behave.
- We are especially interested in safety and lifeness.
- We use the logic TCTL for specification.
- In TCTL we can express properties like:

  "The temperature is always below $20°C$."

- Or

  "If the temperature is above $20°C$ it will get below $20°C$ within 5 seconds."

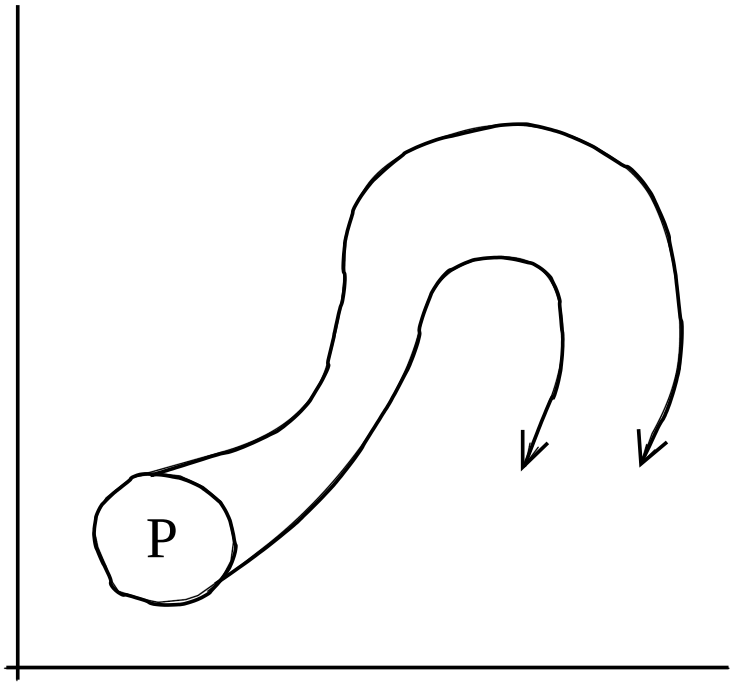
- Or

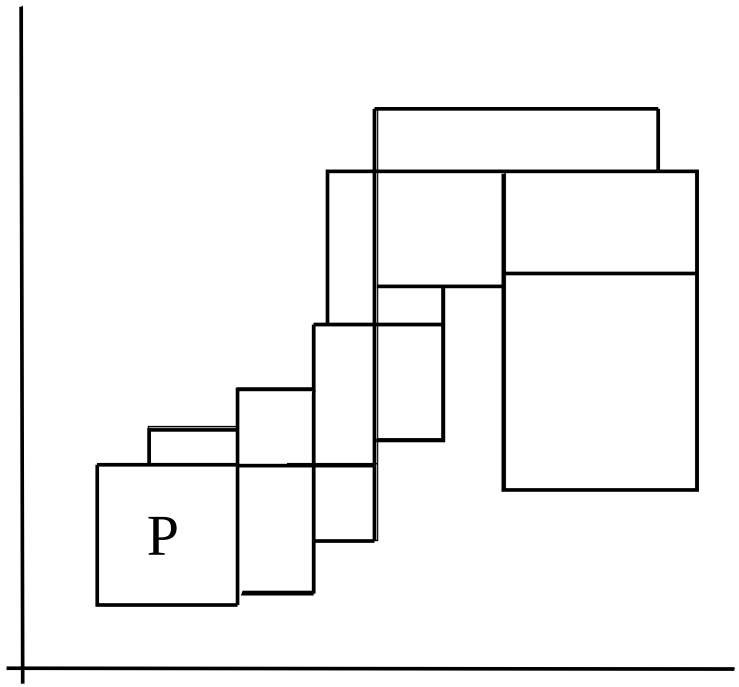
  "It is always the case that the temperature will somewhen in the future get above $20°C$."

The analysis of hybrid systems

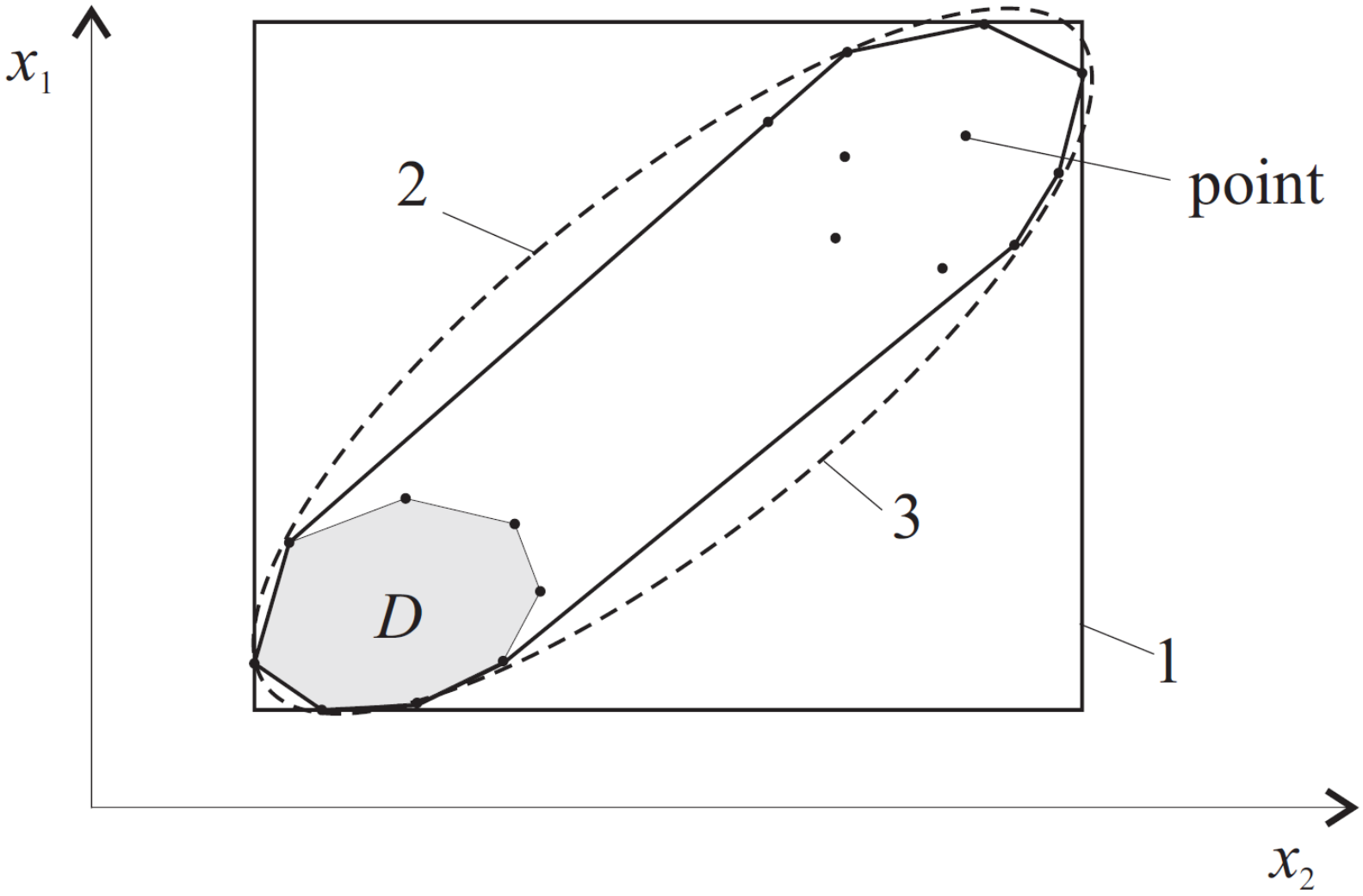
- Assume we modeled a hybrid system as a hybrid automaton.
- Assume we specified a property of the system.
- Can we prove that the system satisfies the property?
- Well, it depends...
- ...on the fact if the logic is decidable for the underlying modeling language.
- We will see for which classes of hybrid automata the general reachability question is decidable.
- We will deal with (unbounded) reachability for timed automata.
- We will deal with bounded reachability for linear hybrid automata.
- We will deal with reachability approximation for hybrid automata.

Reachability approximation for hybrid automata





Motivation



Representation requirements

The geometry should allow efficient computation of the operations for

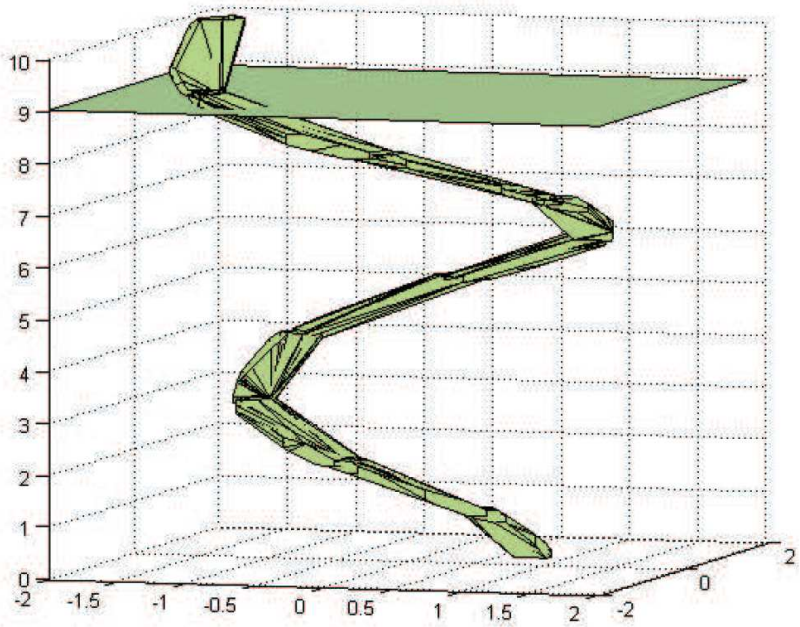
- membership relation,
- union,
- intersection,
- subtraction,
- test for emptiness.

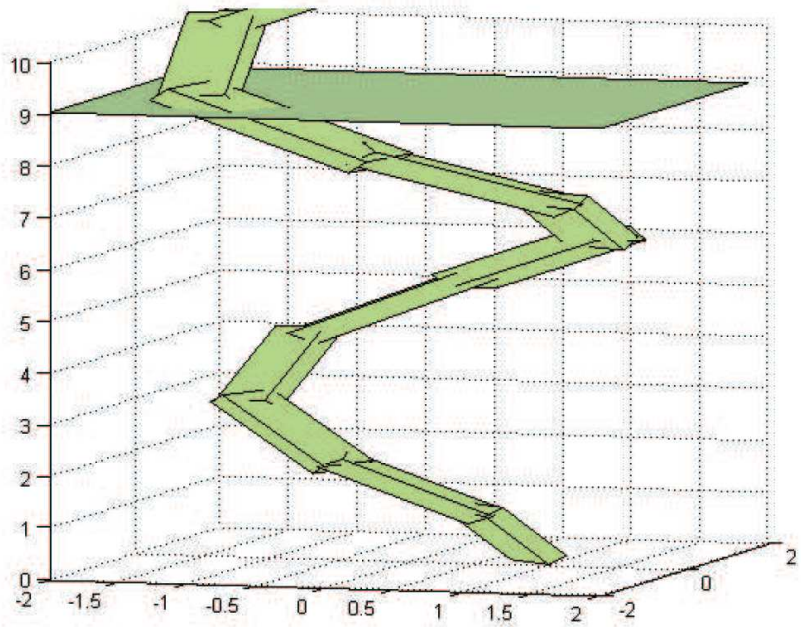
State set representation

Approaches:

- Polyhedra
- Orthogonal polyhedra
- Oriented rectangular hulls
- Zonotopes
- Ellipsoids

Polyhedra (left) and oriented rectangular hulls (right) in reachability computation





Zonotopes in reachability computation