Modeling and analysis of hybrid systems

Introduction

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Informatik 2 - Theory of Hybrid Systems
RWTH Aachen

SS 2010

Organizational

Lecture:

- Thursday 12:30-14:00 in AH I
- Friday 10:45-11:30 in 6019

Exercise:

- Friday 10:00-10:45 in 6019

“Hybrid”

Wikipedia:

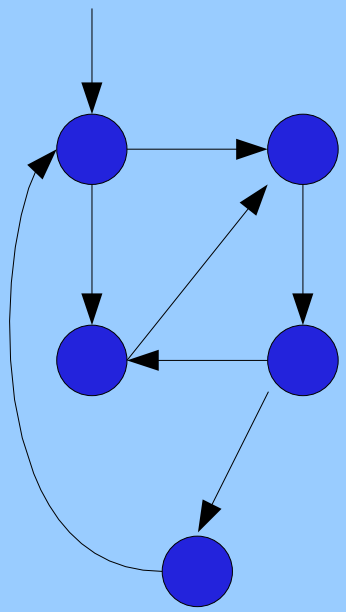
> “A hybrid is the combination of two or more different things, aimed at achieving a particular objective or goal.”

A hybrid rose

A hybrid car

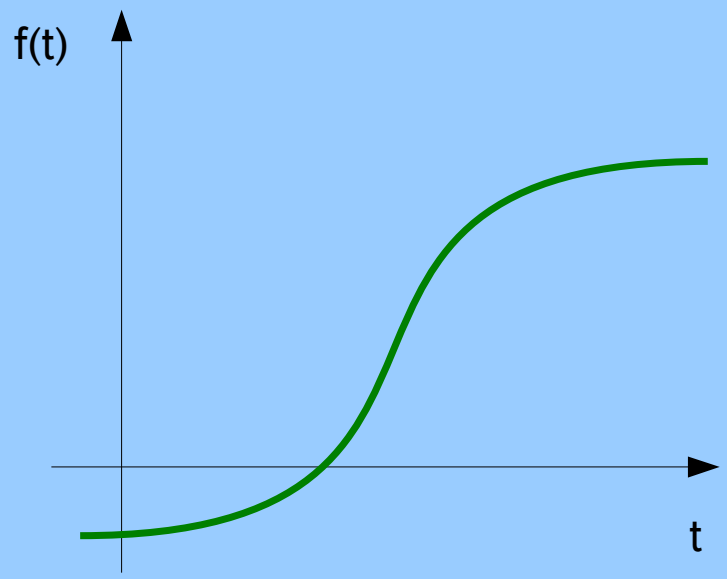
Hybrid in Computer Science

discrete

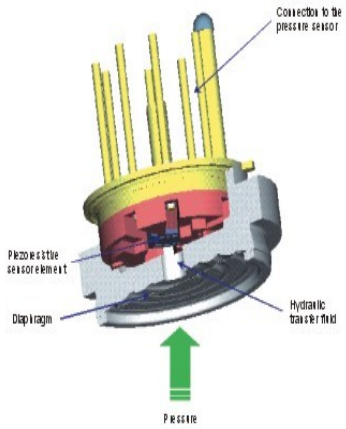


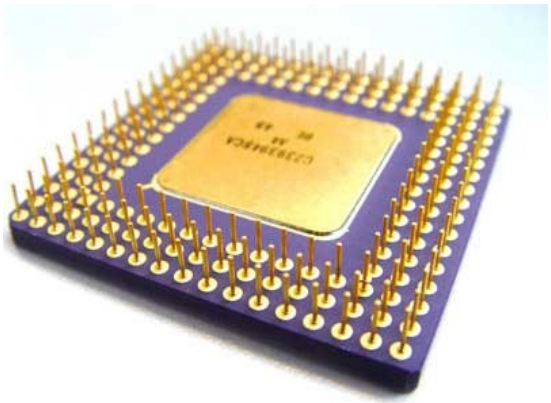
+

continuous



The discrete part



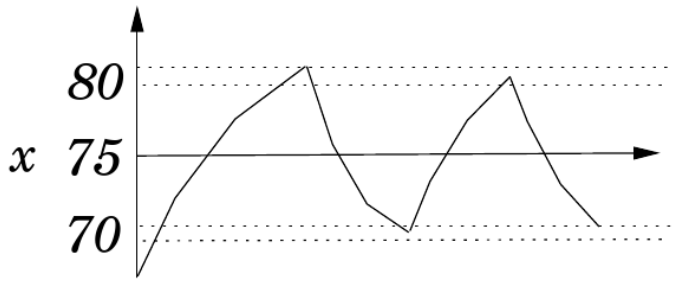


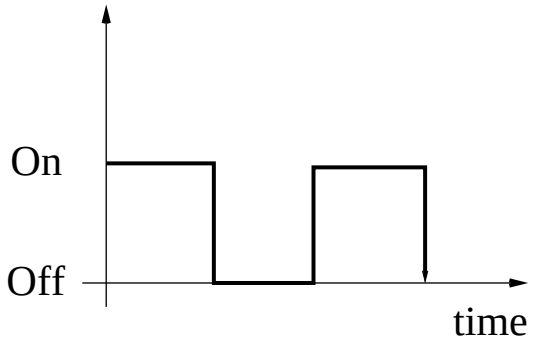
Combined with the continuous part

Example: Thermostat

- temperature x controlled by switching a heater
- x regulated by thermostat:
  - $68° \leq x \leq 70° \rightsquigarrow$ "heater on"
  - $80° \leq x \leq 82° \rightsquigarrow$ "heater off"

Continuous: temperature
Discrete: switching

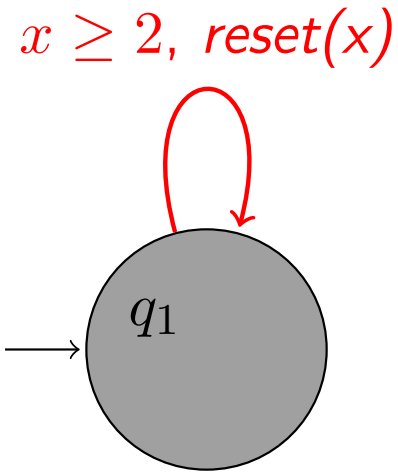


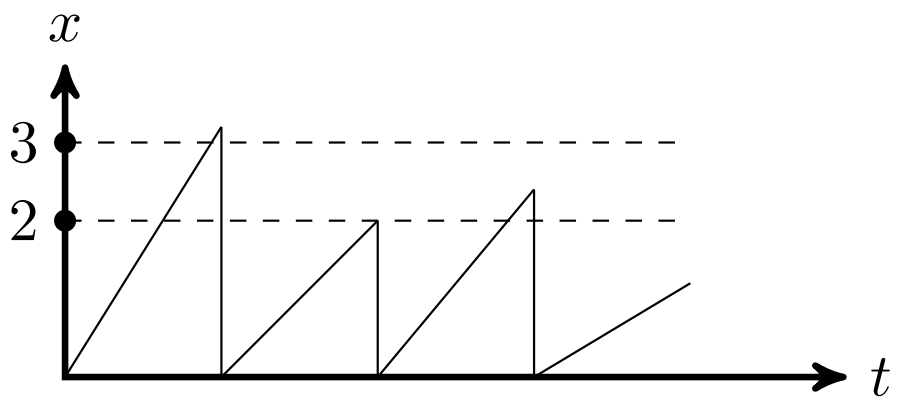


Modeling

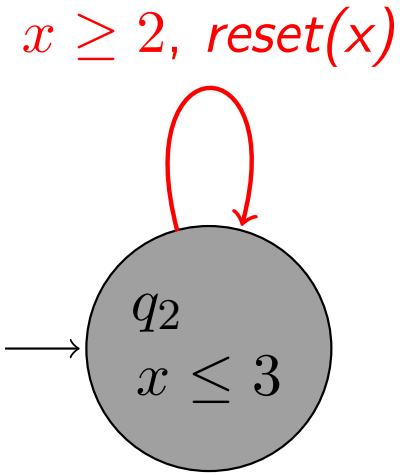
What you perhaps already know: Timed automata

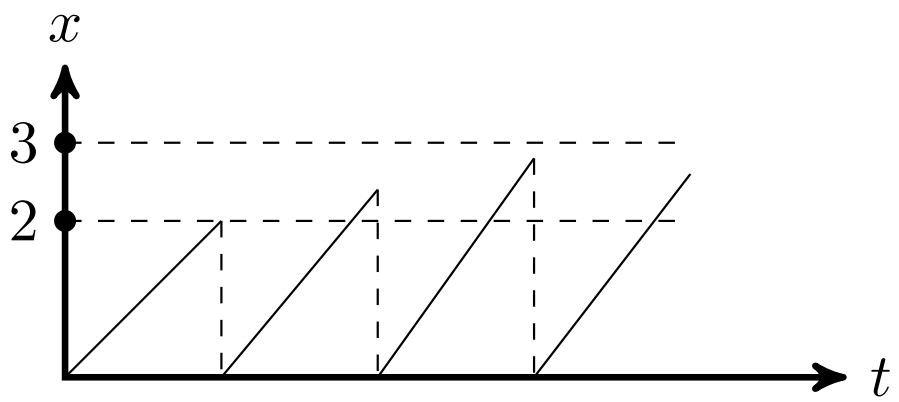
Example: Timed Automaton





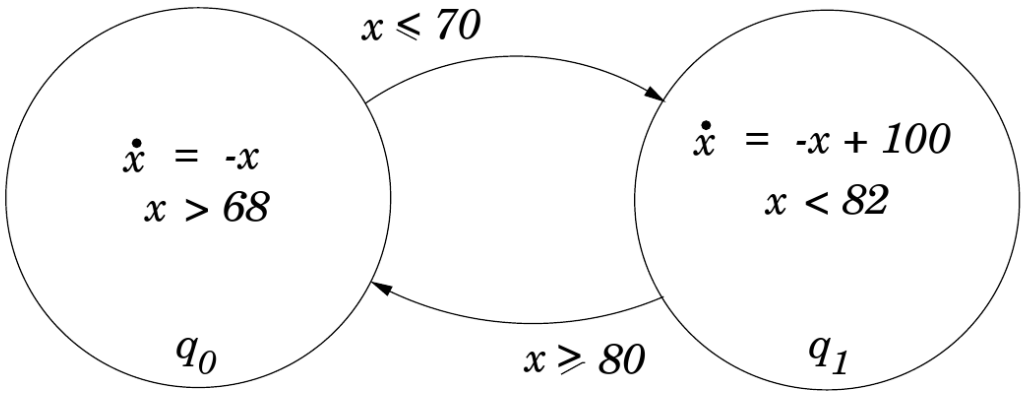
Example: Timed Automaton

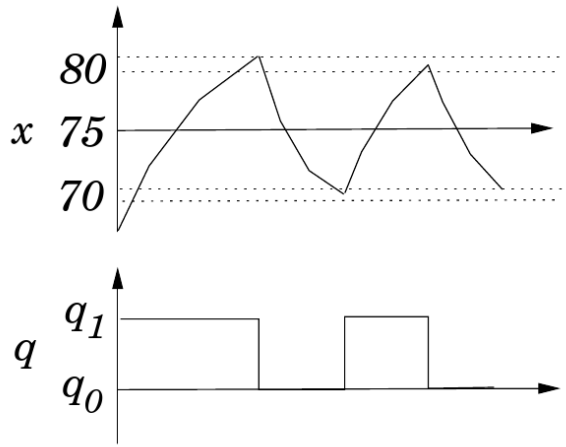




Modeling in general: Hybrid Automata

Let's take again the thermostat as an example.





Logic

- We want to specify how a hybrid system is expected to behave.
- We are especially interested in safety and lifeness.
- We use the logic TCTL for specification.
- In TCTL we can express properties like:

  "The temperature is always below $20°C$."

- Or

  "If the temperature is above $20°C$ it will get below $20°C$ within 5 seconds."

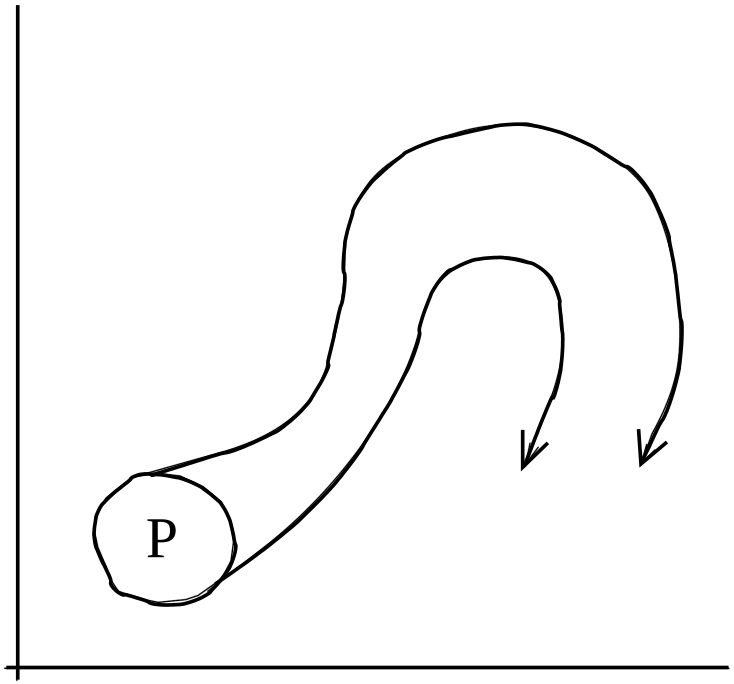
- Or

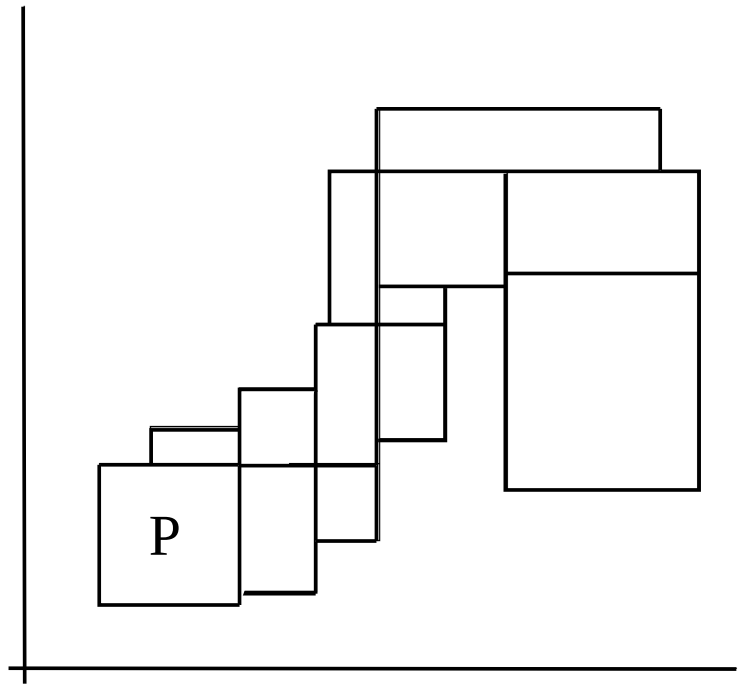
  "It is always the case that the temperature will somewhen in the futute get above $20°C$."

The Analysis of Hybrid Systems

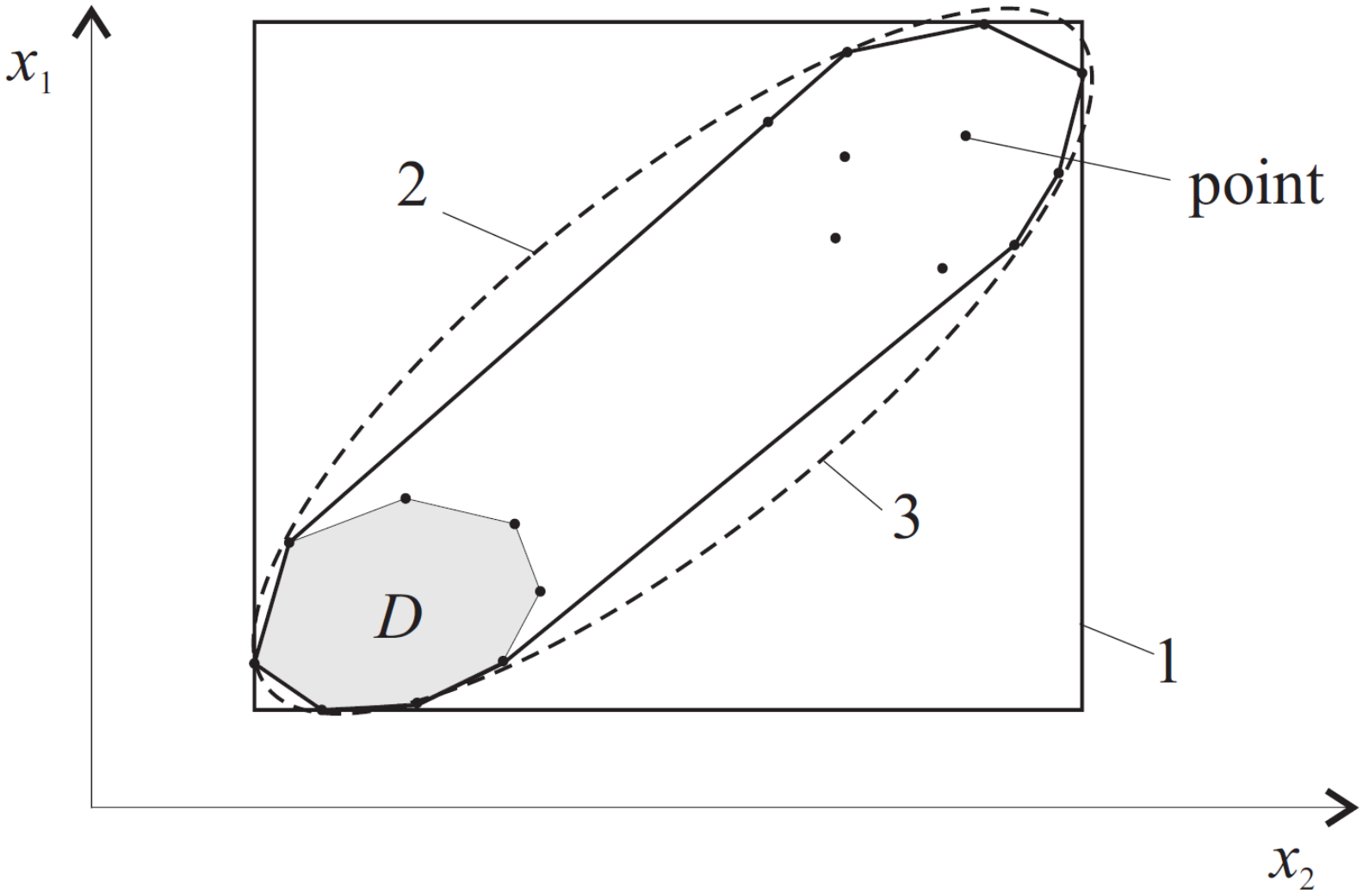
- Assume we modeled a hybrid system as a hybrid automaton.
- Assume we specified a property of the system.
- Can we prove that the system satisfies the property?
- Well, it depends...
- ...on the fact if the logic is decidable for the underlying modeling language.
- We will see for which classes of hybrid automata the general reachability question is decidable.
- We will deal with model checking for timed automata.
- We will see for which classes of hybrid automata the bounded reachability question is decidable.
- We will deal with bounded model checking of linear hybrid automata.

Reachability Approximation for Hybrid Automata





Motivation



Representation Requirements

The geometry should allow efficient computation of the operations for

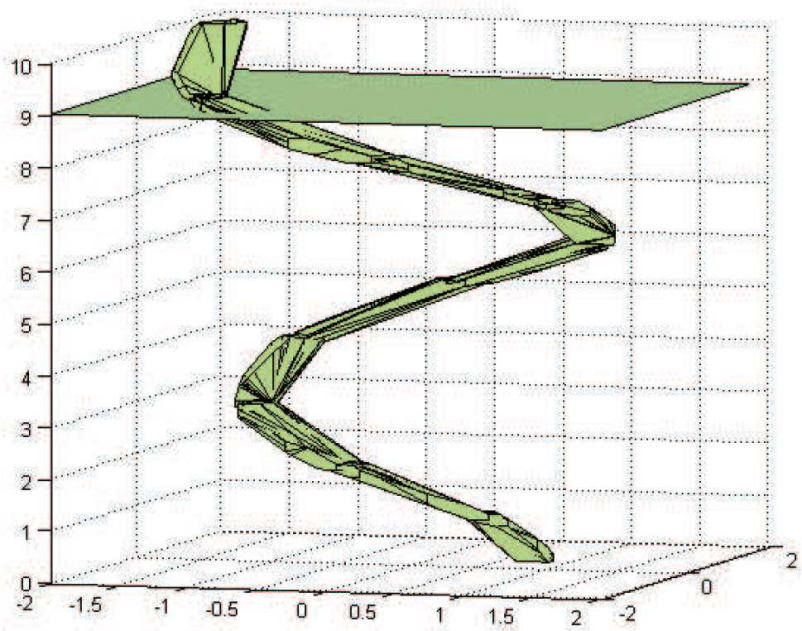
- membership relation,
- union,
- intersection,
- subtraction,
- test for emptiness.

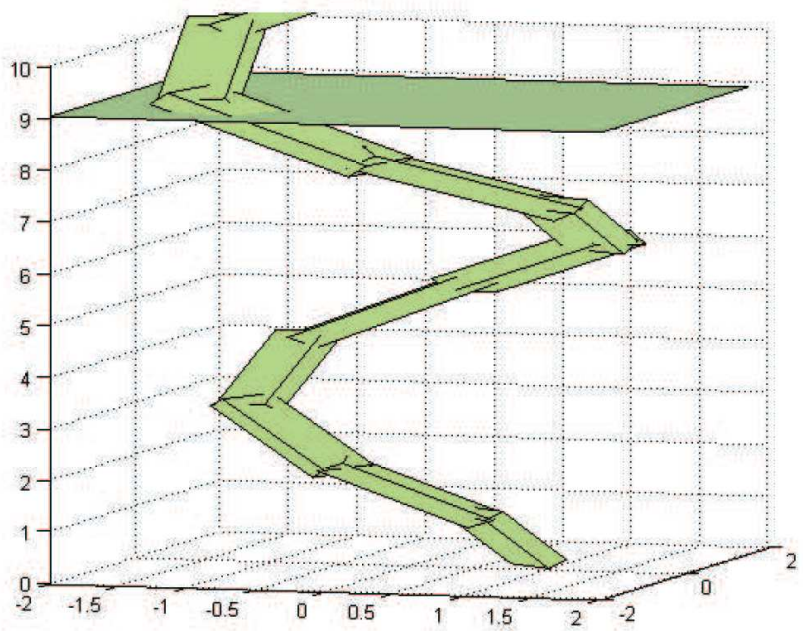
State Set Representation

Approaches:

- Polyhedra
- Orthogonal polyhedra
- Oriented rectangular hulls
- Zonotopes
- Ellipsoids

Oriented Rectangular Hulls in Reachability Computation





Zonotopes in Reachability Computation