# Principles of Model Checking
# Exercise class 3

Linear temporal logic

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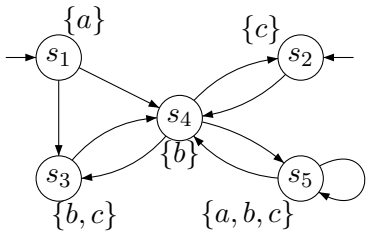
## Problem 1

Consider the following transition system $TS$ over the set of atomic propositions $AP = \{a, b, c\}$:




Decide $TS \models \varphi_i$ for each of the LTL formulas $\varphi_i$ below. Justify your answers! If $TS \not\models \varphi_i$, provide a path $\pi \in \mathit{Paths}(TS)$ such that $\pi \not\models \varphi_i$.

(1) $\varphi_1 = \Box\Diamond c$

(2) $\varphi_2 = \bigcirc\neg c \rightarrow \bigcirc\bigcirc c$

(3) $\varphi_3 = a \mathsf{U} \Box(b \vee c)$

(4) $\varphi_4 = (\bigcirc\bigcirc b) \mathsf{U} (b \vee c)$

## Problem 2

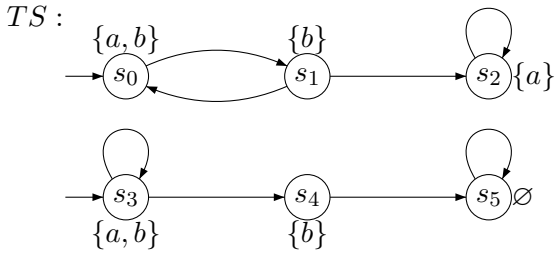
Prove or disprove the following equivalences of LTL formulas:

(1) $\Box\varphi \rightarrow \Diamond\psi \equiv \varphi \mathsf{U} (\psi \vee \neg\varphi)$

(2) $\Box\Diamond\varphi \rightarrow \Box\Diamond\psi \equiv \Box(\varphi \rightarrow \Diamond\psi)$

## Problem 3

Let $AP = \{a, b, c\}$. Consider the transition system $TS$ over $AP$ outlined below




and the LTL fairness assumption

$$\mathit{fair} = (\Box\Diamond\,(a \wedge b) \rightarrow \Box\Diamond\neg c) \wedge (\Diamond\Box\,(a \wedge b) \rightarrow \Box\Diamond\neg b)$$

1. Specify the fair paths of $TS$.
2. Decide for each of the following LTL formulas $\varphi_i$ whether $TS \models_{\mathit{fair}} \varphi_i$:

   $$\varphi_1 = \bigcirc\neg a \rightarrow \Diamond\Box a \qquad \varphi_2 = b\mathsf{U}\Box\neg b \qquad \varphi_3 = b\mathsf{W}\Box\neg b.$$

   In case $TS \not\models_{\mathit{fair}} \varphi_i$, indicate a path $\pi \in \mathit{FairPaths}(TS)$ for which $\pi \not\models \varphi$ holds.

## Problem 4

Consider the LTL-formula $\varphi = a\mathsf{U}\bigcirc a$ over the set of atomic propositions $AP = \{a\}$. Construct an equivalent GNBA $\mathcal{G}$ (i.e. $\mathcal{L}_\omega(\mathcal{G}) = \mathit{Words}(\varphi)$) according to the algorithm given in the proof of Theorem 5.37.