Principles of Model Checking
Solutions to exercise class 6

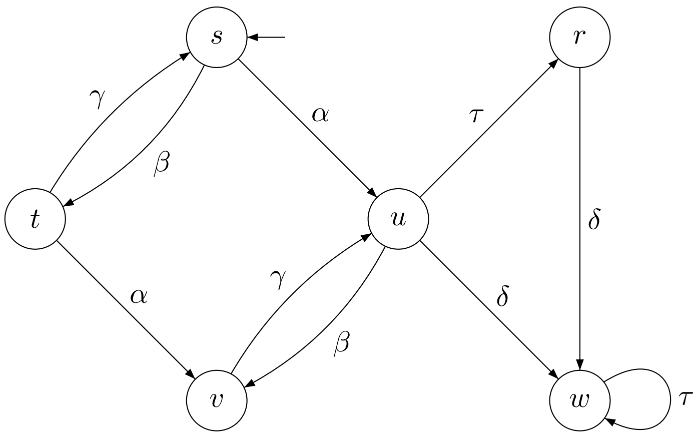
Partial Order Reduction

Prof. Dr. Joost-Pieter Katoen, Dr. Taolue Chen,
Dr. Jeroen Ketema, and Ir. Mark Timmer

October, 19, 2012

Problem 1

Consider the transition system TS below, with action set $Act = \{\alpha, \beta, \gamma, \delta, \tau\}$. Determine the pairs of independent actions.



Solution:

Recall that actions α_1 and α_2 are independent if for any $s \in S$ with $\alpha_1, \alpha_2 \in Act(s)$ it holds that

$$\alpha_1 \in Act(\alpha_2(s)) \quad \wedge \quad \alpha_2 \in Act(\alpha_1(s)) \quad \wedge \quad \alpha_1(\alpha_2(s)) = \alpha_2(\alpha_1(s)).$$

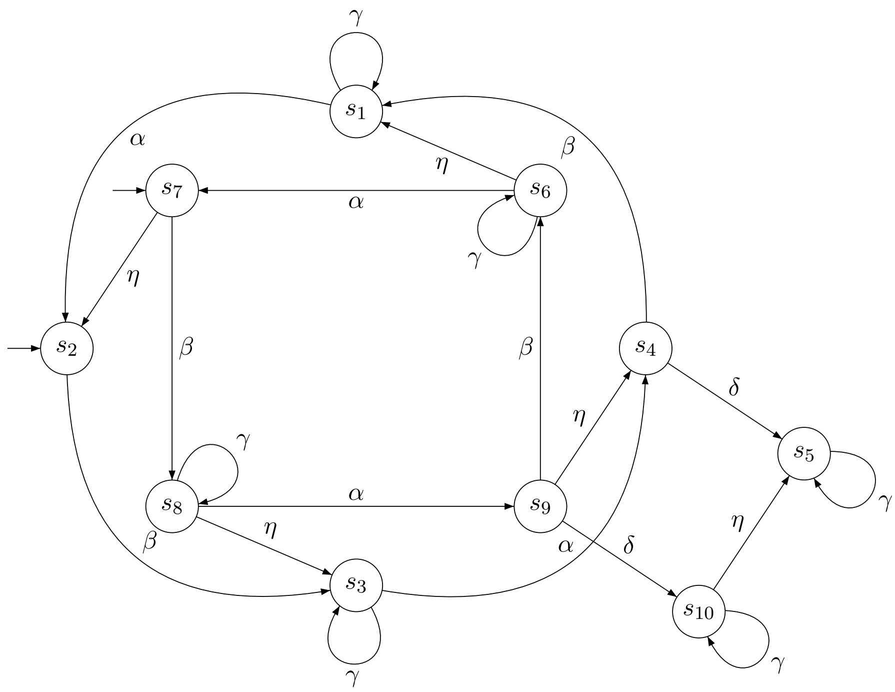
Note that if α_1 and α_2 are never enabled in the same state, it is obvious that they are independent (indicated below by $+^*$).

action pair	$\alpha\beta$	$\alpha\gamma$	$\alpha\delta$	$\alpha\tau$	$\beta\gamma$	$\beta\delta$	$\beta\tau$	$\gamma\delta$	$\gamma\tau$	$\delta\tau$
independent	$+$	$+$	$+^*$	$+^*$	$+^*$	$-$	$-$	$+^*$	$+^*$	$+$

The reason for the two minuses can be explored in state u.

Problem 2

Consider the transition system below:



The states labeling is as follows:

- $L(s_{10}) = \varnothing$
- $L(s_6) = L(s_7) = \{a\}$
- $L(s_3) = L(s_4) = L(s_5) = L(s_8) = L(s_9) = \{b\}$
- $L(s_1) = L(s_2) = \{a, b\}$

Prove or disprove that each of the following *ample sets* satisfy requirements *A1* through *A3*. Also check whether the requirement *A4* holds.

- $\textit{ample}(s_6) = \{\alpha, \gamma\}$
- $\textit{ample}(s_7) = \{\beta\}$
- $\textit{ample}(s_8) = \{\alpha\}$
- $\textit{ample}(s_9) = \{\alpha, \beta, \delta\}$
- $\textit{ample}(s_{10}) = \{\gamma, \eta\}$

In case some of the conditions *A1* through *A4* do not hold, modify the *ample sets* in an appropriate way to fix it. Clarify your changes.

Solution:

- Let us first indicate dependent and independent actions which we might need:
  - Action η is independent of $\alpha, \beta, \gamma, \delta$.
  - Action α and γ are dependent, and so are δ and β.
- Now let's indicate the stutter actions:
  - Clearly α, γ are stutter actions.
- Considering all given *ample sets*, and the condition $(A4)$, we realize that for the cycle s_6, s_7, s_8, s_9, s_6, $\eta \in Act(s_6)$, but $\eta \notin ample(s_6) \cup ample(s_7) \cup ample(s_8) \cup ample(s_9)$. Thus, condition $(A4)$ is violated. In order to fix it we have to add η for example to $ample(s_7)$. Later, we will see that η has to be added to $ample(s_7)$ anyway.

  On the cycle s_2, s_3, s_4, s_1, s_2 all states are fully-expanded, so they clearly satisfy the cycle condition.
- Consider each of the given *ample sets* (+ stands for satisfied, − for not satisfied):
  - $ample(s_6) = \{\alpha, \gamma\}$:

    (A1) +: $ample(s_6) \neq \varnothing$.

    (A2) +: All paths starting from s_6 either start with an ample action α, γ, or with η. Since η is independent of both α and γ, this is fine. After η, either γ or α is observed. Hence, every path from s_6 at some point reaches an ample action while beforehand only containing actions independent of $ample(s_6)$.

    (A3) +: as α and γ are stutter actions, (A3) clearly holds.

  - $ample(s_7) = \{\beta\}$:

    (A1) +: $ample(s_7) \neq \varnothing$.

    (A2) +: All paths from s_7 start by either β or $\eta\,\beta$. Since β is in the ample set, and η is independent of β, (A2) is satisfied.

    (A3) −: $ample(s_7) \neq Act(s_7)$ and β is not a stutter action, so we should have $ample(s_7) = \{\beta, \eta\}$.

- $ample(s_8) = \{\alpha\}$:
  - (A1) +: $ample(s_8) \neq \varnothing$.
  - (A2) −: For the execution $s_8 \xrightarrow{\gamma} s_8$ we find that $ample(s_8)$ and γ are dependent, so we have to have $ample(s_8) = \{\alpha, \gamma\}$. Then, (A2) indeed holds, as all paths starting from s_8 either start with ample action γ or α, or by $\eta\,\alpha$ (which is also fine since η is independent of $ample(s_8)$ and α is an ample action).
  - (A3) +: as α is a stutter action (for $ample(s_8) = \{\alpha, \gamma\}$ the condition holds as well, since γ is also a stutter action).

- $ample(s_9) = \{\alpha, \beta, \delta\}$:
  - (A1) −: $ample(s_9) \neq \varnothing$ but $ample(s_9) \setminus Act(s_9) = \{\alpha\}$, thus it should be $ample(s_9) = \{\beta, \delta\}$.
  - (A2) +: All paths starting from s_9 begin with an ample action, or with the independent action η followed by an ample action.
  - (A3) −: As $ample(s_9) \neq Act(s_9)$ and β, δ are not stutter actions, (A3) is violated. Thus to fix it, we are obliged to set $ample(s_9) = \{\beta, \delta, \eta\}$, note that (A1), (A2), (A3) then hold, because $ample(s_9) = Act(s_9)$.

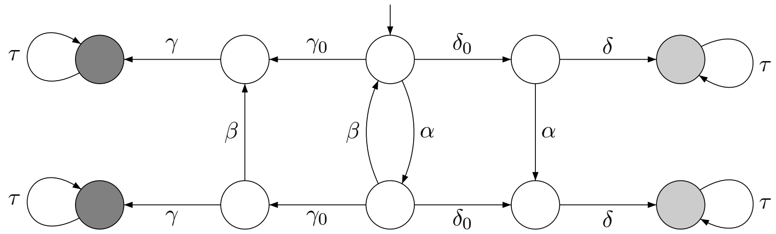
- $ample(s_{10}) = \{\gamma, \eta\}$:
  - (A1) +: $ample(s_{10}) \neq \varnothing$.
  - (A2) +: $ample(s_{10}) = Act(s_{10})$.
  - (A3) +: $ample(s_{10}) = Act(s_{10})$.

  Note that we can not have $ample(s_{10}) = \{\gamma\}$, because it will violate (A4).

• After fixing all the problems with the given *ample sets* we have:
  - $ample(s_6) = \{\alpha, \gamma\}$
  - $ample(s_7) = \{\beta, \eta\}$
  - $ample(s_8) = \{\alpha, \gamma\}$
  - $ample(s_9) = \{\beta, \delta, \eta\}$
  - $ample(s_{10}) = \{\gamma, \eta\}$

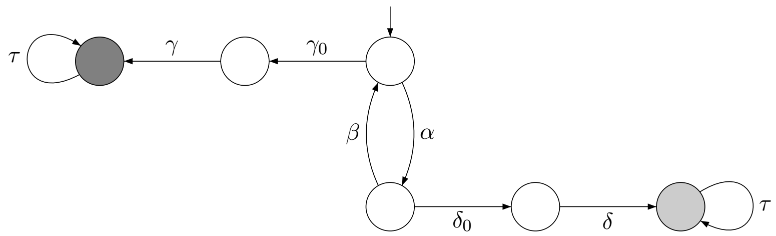
Problem 3

Consider the transition system TS shown in the following figure. Show that the ample set conditions (A1)-(A4) do not allow for any state reduction, although there is a smaller subsystem $\widehat{TS}$ that is stutter-trace equivalent to TS.



Solution:

The reduced system shown in the following picture is easily seen to be stutter-trace equivalent to TS:



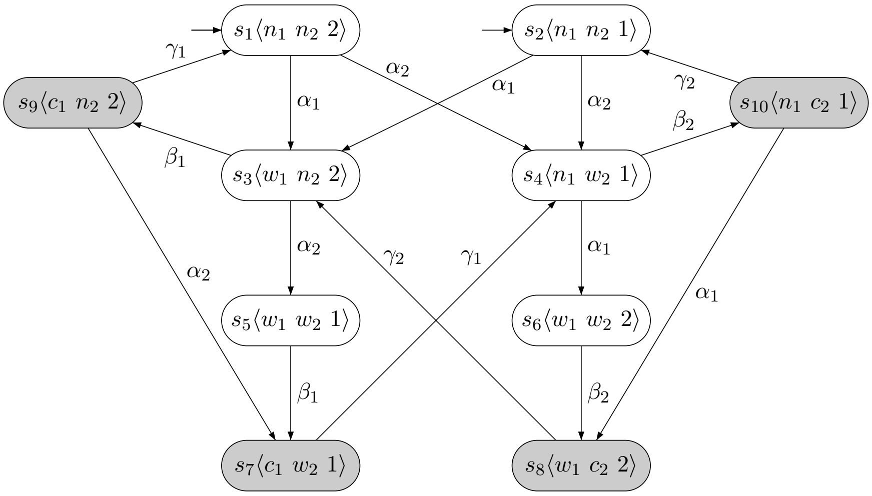
Conditions (A1)-(A4) do not allow for any state reduction, though, since the following pairs of actions are dependent:

$$(\alpha, \gamma_0),\ (\beta, \delta_0),\ (\gamma_0, \delta_0),\ (\gamma, \beta),\ (\alpha, \delta).$$

Thus, the initial state s_0 has to be fully expanded by the dependency condition (A2). The same argument applies to $\alpha(s_0)$ which has to be fully expanded too. Since β and γ are dependent, the γ_0-successor of $\alpha(s_0)$ has to be fully expanded. The same holds for the δ_0-successor of s_0, since α and δ are dependent. The nonemptiness condition (A1) then yields that all states are fully expanded.

Problem 4

Consider the transition system TS_{Pet} below, which is for the Peterson mutual exclusion algorithm.



Questions:

(a) Which actions are independent?

(b) Apply the partial order reduction approach to get a reduced system equivalent to TS_{Pet}.

Solution:

(a) In state s_1 and s_2 it can be seen that α_1 and α_2 are dependent. It is easy to verify that in no other state any dependent actions occur, so except for (α_1, α_2) all action pairs are independent.

(b) In s_1 and s_2 no reduction is possible, since α_1 and α_2 are dependent. In s_3 it is not possible to take $ample(s_3) = \{\beta_1\}$ since β_1 is not a stutter action, and it is also not possible to take $ample(s_3) = \{\alpha_2\}$ since in that case there would be a path $s_3 \xrightarrow{\beta_1} s_9 \xrightarrow{\gamma_1} s_1 \xrightarrow{\alpha_1} s_3$ violating (A2), since α_1 depends on α_2. Hence, also s_3 should be fully expanded. For s_4, s_9 and s_{10} similar arguments can be made. In s_5, s_6, s_7 and s_8 there is only one transition, so also there no reduction is possible.

Hence, $ample(s_i) = Act(s_i)$ for every state s_i, and the reduced system equals the original system.