

Introduction to Model Checking

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– Series 10 –

Hand in on June 22 before the exercise class.

Exercise 1

(3 + 1 points)

Consider the fragment ECTL of CTL which consists of formulas built according to the following grammar:

$$\begin{aligned}\Phi ::= & a \mid \neg a \mid \Phi \wedge \Phi \mid \exists \varphi \\ \varphi ::= & \bigcirc \Phi \mid \square \Phi \mid \Phi \mathbf{U} \Phi\end{aligned}$$

Therefore, ECTL-formulas are built by atomic propositions, negated atomic propositions, the boolean connective \wedge and the path quantifier \exists together with the modalities \bigcirc , \square and \mathbf{U} .

For two transition systems $TS_1 = (S_1, Act, \rightarrow_1, I_1, AP, L_1)$ and $TS_2 = (S_2, Act, \rightarrow_2, I_2, AP, L_2)$, we define $TS_1 \subseteq TS_2$ iff $S_1 \subseteq S_2$, $\rightarrow_1 \subseteq \rightarrow_2$, $I_1 = I_2$ and $L_1(s) = L_2(s)$ for all $s \in S_1$.

(a) Prove, that for all ECTL-formulas Φ and all transition systems TS_1, TS_2 with $TS_1 \subseteq TS_2$, it holds:

$$TS_1 \models \Phi \implies TS_2 \models \Phi.$$

(b) Give a CTL-formula which is not equivalent to any other ECTL-formula. Justify your answer!

Exercise 2

(1 points)

Transform the CTL-formula $\Phi = \neg \forall \diamond (\forall (\forall \square b) \mathbf{U} (\forall \bigcirc a))$ into an equivalent CTL-formula in existential normal form.

Exercise 3

(2 + 2 points)

We consider again the incomparable expressiveness of CTL and LTL.

(a) Using Theorem 6.18, prove that there does not exist an equivalent LTL-formula for the CTL-formula $\Phi_1 = \forall \diamond (a \wedge \exists \bigcirc a)$.

(b) Now prove directly (i.e. without Theorem 6.18), that there does not exist an equivalent LTL-formula for the CTL-formula $\Phi_2 = \forall \diamond \exists \bigcirc \forall \diamond \neg a$.

Hint: Argument by contraposition!

Exercise 4

(2 + 3 points)

Consider the following CTL-formulas

$$\Phi_1 = \exists \diamond \forall \square c \quad \text{and} \quad \Phi_2 = \forall (a \mathbf{U} \forall \diamond c)$$

and the transition system outlined on the right. Decide whether $TS \models \Phi_i$ for $i = 1, 2$ using the CTL model checking algorithm from the lecture. Sketch its main steps!

