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Introduction to Model Checking
Summer term 2007

– Series 4 –

Hand in on May 4 before the exercise class.

Exercise 1 (3 points)

Let P denote the set of traces of the form $\sigma = A_0 A_1 A_2 \cdots \in \left(2^{AP}\right)^\omega$ such that

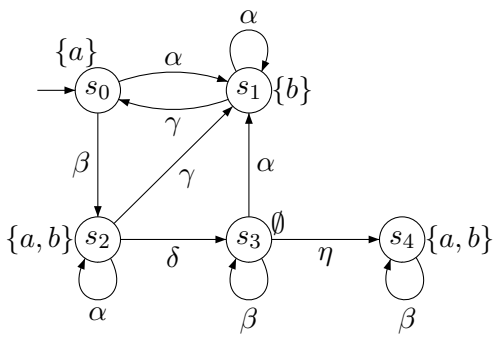
$$\overset{\infty}{\exists}\, k.\ A_k = \{a, b\} \quad \wedge \quad \exists n \geq 0.\ \forall k > n.\ \big(a \in A_k \Rightarrow b \in A_{k+1}\big).$$

Consider the following fairness assumptions with respect to the transition system TS outlined on the right:

a) $\mathcal{F}_1 = \big(\{\{\alpha\}\}, \{\{\beta\}, \{\delta, \gamma\}, \{\eta\}\}, \emptyset\big)$.
Decide whether $TS \models_{\mathcal{F}_1} P$.

b) $\mathcal{F}_2 = \Big(\{\{\alpha\}\}, \{\{\beta\}, \{\gamma\}\}, \{\{\eta\}\}\Big)$.
Decide whether $TS \models_{\mathcal{F}_2} P$.

Justify your answers!



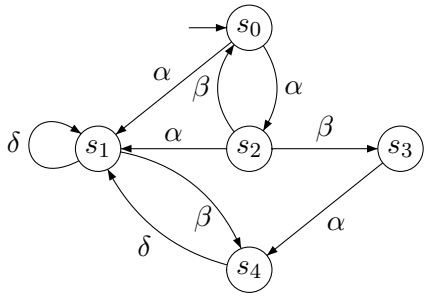
Exercise 2 (3 points)

Consider the transition system TS on the right (where atomic propositions are omitted). Decide which of the following fairness assumptions $\mathcal{F}_i$ are realizable for TS. Justify your answers!

a) $\mathcal{F}_1 = (\{\{\alpha\}\}, \{\{\delta\}\}, \{\{\alpha, \beta\}\})$

b) $\mathcal{F}_2 = (\{\{\delta, \alpha\}\}, \{\{\alpha, \beta\}\}, \{\{\delta\}\})$

c) $\mathcal{F}_3 = (\{\{\alpha, \delta\}, \{\beta\}\}, \{\{\alpha, \beta\}\}, \{\{\delta\}\})$



Exercise 3 (1 + 1 points)

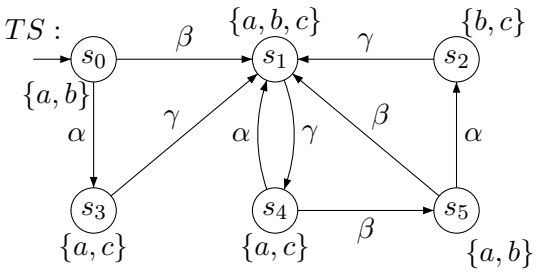
Let $n \geq 1$. Consider the language $L_n \subseteq \Sigma^*$ over the alphabet $\Sigma = \{A, B\}$ that consists of all finite words where the symbol B is on position n from the right, i.e., L_n contains exactly the words $A_1 A_2 \ldots A_k \in \{A, B\}^*$ where $k \geq n$ and $A_{k-n+1} = B$. For instance, the word $ABBAABAB$ is in L_3.

a) Construct an NFA $\mathcal{A}_n$ with at most $n + 1$ states such that $\mathcal{L}(\mathcal{A}_n) = L_n$.

b) Determinize this NFA $\mathcal{A}_n$ using the powerset construction algorithm.

Exercise 4 **(2 + 2 points)**

Consider the following transition system TS



and the regular safety property

$$P_{safe} = \begin{array}{l}\text{“always if } a \text{ is valid and } b \wedge \neg c \text{ was valid somewhere before,}\\ \text{then } a \text{ and } b \text{ do not hold thereafter at least until } c \text{ holds”}\end{array}$$

As an example, it holds:

$$\begin{aligned}\{b\}\emptyset\{a,b\}\{a,b,c\} &\in pref(P_{safe})\\ \{a,b\}\{a,b\}\emptyset\{b,c\} &\in pref(P_{safe})\\ \{b\}\{a,c\}\{a\}\{a,b,c\} &\in BadPref(P_{safe})\\ \{b\}\{a,c\}\{a,c\}\{a\} &\in BadPref(P_{safe})\end{aligned}$$

Questions:

(a) Define an NFA $\mathcal{A}$ such that $\mathcal{L}(\mathcal{A}) = MinBadPref(P_{safe})$.

(b) Decide whether $TS \models P_{safe}$ using the $TS \otimes \mathcal{A}$ construction.
Provide a counterexample if $TS \not\models P_{safe}$.