

## Introduction to Model Checking

### Summer term 2007

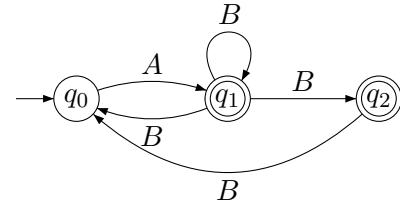
#### – Series 6 –

Hand in on May 18 before the exercise class.

#### Exercise 1

(2 points)

Consider the GNBA outlined on the right with acceptance sets  $F_1 = \{q_1\}$  and  $F_2 = \{q_2\}$ . Construct an equivalent NBA using the transformation introduced in the lecture.



#### Exercise 2

(3 points)

Provide NBA  $\mathcal{A}_1$  and  $\mathcal{A}_2$  for the languages given by the expressions  $(AC + B)^*B^\omega$  and  $(B^*AC)^\omega$  and apply the product construction (using GNBA) to obtain an NBA  $\mathcal{A}$  with  $\mathcal{L}_\omega(\mathcal{A}) = \mathcal{L}_\omega(\mathcal{A}_1) \cap \mathcal{L}_\omega(\mathcal{A}_2)$ . Justify, why  $\mathcal{L}_\omega(\mathcal{G}) = \emptyset$  where  $\mathcal{G}$  denotes the GNBA accepting the intersection.

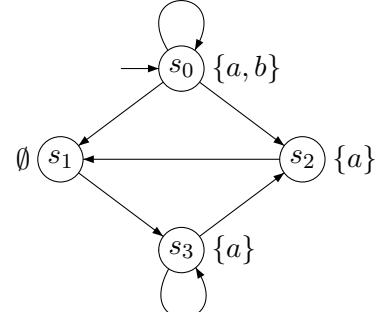
#### Exercise 3

(1 + 2 + 2 + 1 points)

We consider model checking of  $\omega$ -regular LT properties which are defined by LTL formulas. Therefore let  $\varphi_1$  and  $\varphi_2$  be as follows:

$$\begin{aligned}\varphi_1 &= \square \diamond a \rightarrow \square \diamond b \quad \text{and} \\ \varphi_2 &= \diamond(a \wedge \bigcirc a).\end{aligned}$$

Further, our model is represented by the transition system  $TS$  over  $AP = \{a, b\}$  which is given as outlined on the right. We check whether  $TS \models \varphi_i$  for  $i = 1, 2$  using the nested depth-first search algorithm from the lecture. Therefore proceed as follows:

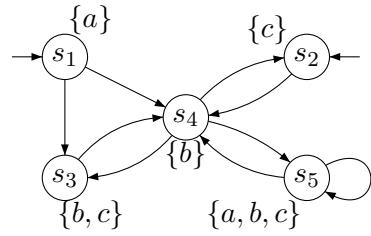


- Derive an NBA  $\mathcal{A}_i$  for the LTL formula  $\neg\varphi_i$  (for  $i = 1, 2$ ).  
More precisely, for  $\mathcal{A}_i$  it must hold  $\mathcal{L}_\omega(\mathcal{A}_i) = \mathcal{L}_\omega(\neg\varphi_i)$ .  
*Hint: Four, respectively three states suffice.*
- Outline the reachable fragment of the product transition system  $TS \otimes \mathcal{A}_i$ .
- Sketch the main steps of the nested depth-first search algorithm for the persistency check on  $TS \otimes \mathcal{A}_i$ .
- Provide the counterexample computed by the algorithm if  $TS \not\models \varphi_i$ .

### Exercise 4

(3 points)

Consider the transition system  $TS$  over the set of atomic propositions  $AP = \{a, b, c\}$ :



Decide for each of the LTL formulas  $\varphi_i$  below, whether  $TS \models \varphi_i$  holds. Justify your answers!  
 If  $TS \not\models \varphi_i$ , provide a path  $\pi \in Paths(TS)$  such that  $\pi \not\models \varphi_i$ .

$$\varphi_1 = \Diamond \Box c$$

$$\varphi_4 = \Box a$$

$$\varphi_2 = \Box \Diamond c$$

$$\varphi_5 = a \mathsf{U} \Box(b \vee c)$$

$$\varphi_3 = \bigcirc \neg c \rightarrow \bigcirc \bigcirc c$$

$$\varphi_6 = (\bigcirc \bigcirc b) \mathsf{U}(b \vee c)$$