RHEINISCH-WESTFÄLISCHE TECHNISCHE HOCHSCHULE AACHEN

LEHRSTUHL FÜR INFORMATIK II
RWTH Aachen · D-52056 Aachen
http://www-i2.informatik.rwth-aachen.de/

Prof. Dr. Ir. J.-P. Katoen
M. Neuhäußer

Introduction to Model Checking
Summer term 2007

– Series 8 –

Hand in on June 8 before the exercise class.

Exercise 1 (2 + 2 points)

We consider the release operator R which is defined as $\varphi \mathsf{R} \psi := \neg(\neg\varphi \mathsf{U} \neg\psi)$.

a) Informally describe the meaning of the expansion law for the release operator R. Then prove its correctness formally.

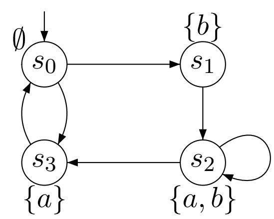
b) Prove the following two equivalence laws that express R by W and vice versa:

- $\varphi \mathsf{R} \psi \equiv (\neg\varphi \wedge \psi)\, \mathsf{W}\, (\varphi \wedge \psi)$
- $\varphi \mathsf{W} \psi \equiv (\neg\varphi \vee \psi)\, \mathsf{R}\, (\varphi \vee \psi)$

Exercise 2 (0.5 + 1.5 + 2 + 1 + 1 + 2 points)

We consider the LTL formula $\varphi = \Box(a \rightarrow (\neg b \mathsf{U}(a \wedge b)))$ over the set $AP = \{a, b\}$ of atomic propositions and want to check $TS \models \varphi$ wrt. the transition system outlined on the right.

TS :



a) To check $TS \models \varphi$, convert $\neg\varphi$ into an equivalent LTL-formula ψ which is constructed according to the following grammar:

$$\Phi ::= true \mid false \mid a \mid b \mid \Phi \wedge \Phi \mid \neg\Phi \mid \bigcirc \Phi \mid \Phi \mathsf{U} \Phi.$$

Then construct $closure(\psi)$.

b) Give the elementary sets wrt. $closure(\psi)$!

c) Construct the GNBA $\mathcal{G}_\psi$ by providing its initial states, its acceptance set and its transition relation. Use the algorithm given in the lecture.
Hint: It suffices to provide the transition relation as a table.

d) Now, construct an NBA $\mathcal{A}_{\neg\varphi}$ **directly** from $\neg\varphi$, i.e. without relying on $\mathcal{G}_\psi$.
Hint: Four states suffice!

e) Construct $TS \otimes \mathcal{A}_{\neg\varphi}$.

f) Use the Nested DFS algorithm from the lecture to check $TS \models \varphi$. Therefore sketch the algorithm's main steps and interpret its outcome!

Exercise 3 (2 points)

Let φ be an LTL-formula over a set of atomic propositions AP. Prove the following property:
For all elementary sets $B \subseteq closure(\varphi)$ and for all $B' \in \delta(B, B \cap AP)$, it holds:

$$\neg \bigcirc \psi \in B \iff \psi \notin B'.$$