

## Introduction to Model Checking Summer term 2007

### – Series 9 –

Hand in on June 15 before the exercise class.

#### Exercise 1

(4 points)

Express the following properties as CTL formulas over  $AP = \{a, b, c\}$  and provide a justification. For more complicated formulas, also comment on their subformulas!

- There exists a path on which for every state  $s$  it holds: there exists a path which starts in  $s$  and on which eventually  $a$  holds and in the next state,  $\neg a$  holds.
- There exists a state  $s$  for which it holds:  $a$  is true and on all paths starting from  $s$ ,  $c$  holds as long as  $b$  does not hold.
- On every path it holds for every state:  $a$  is valid if and only if  $b$  is valid and in the previous state,  $c$  is valid. *Hint:* Note that this excludes the case where  $a$  is valid at the beginning.

#### Exercise 2

(2 points)

Provide two finite transition systems  $TS_1$  and  $TS_2$  (without terminal states, and over the same set of atomic propositions) and a CTL formula  $\Phi$  such that  $Traces(TS_1) = Traces(TS_2)$  and  $TS_1 \models \Phi$ , but  $TS_2 \not\models \Phi$ .

#### Exercise 3

(4 points)

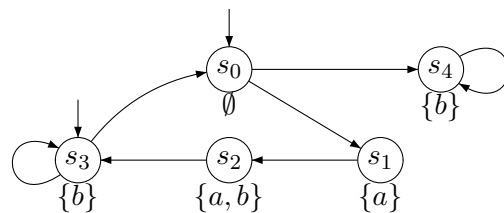
Consider the following CTL formulas and the transition system  $TS$  outlined on the right:

$$\Phi_1 = \forall(aUb) \vee \exists\bigcirc(\forall\Box b)$$

$$\Phi_2 = \forall\Box\forall(aUb)$$

$$\Phi_3 = (a \wedge b) \rightarrow \exists\Box\exists\bigcirc\forall(bWa)$$

$$\Phi_4 = (\forall\Box\exists\Diamond\Phi_3)$$



Give the satisfaction sets  $Sat(\Phi_i)$  and decide whether  $TS \models \Phi_i$  holds ( $1 \leq i \leq 4$ ).

#### Exercise 4

(2 + 2 points)

Prove or disprove the following implications:

- Let  $\Phi_1 = \forall\Diamond a \vee \forall\Diamond b$  and  $\Phi_2 = \forall\Diamond(a \vee b)$ .  
Prove or disprove the following implications:  $\Phi_1 \implies \Phi_2$  and  $\Phi_2 \implies \Phi_1$ .
- Now consider  $\Psi_1 = \exists(aU\exists(bUc))$  and  $\Psi_2 = \exists(\exists(aUb)Uc)$ .  
Again, prove or disprove  $\Psi_1 \implies \Psi_2$  and  $\Psi_2 \implies \Psi_1$ .