Channel Systems

Lecture #4 of Model Checking

Joost-Pieter Katoen

Lehrstuhl 2: Software Modeling and Verification

E-mail: `katoen@cs.rwth-aachen.de`

April 11, 2007

Overview Lecture #4

- *Concurrency*
  - The interleaving paradigm
- Communication principles
  - Shared variable "communication"
  - Handshaking
  - Synchronous communication
- ⇒ Channel systems
  - nanoPromela
- The state-space explosion problem

Channels

- Processes communicate via *channels* ($c \in$ *Chan*)
- Channels are first-in, first-out buffers
- Channels are types (wrt. their content — *dom*(c))
- Channels buffer messages (of appropriate type)
- Channel capacity = maximum $\#$ messages that can be stored
  - if *cap*$(c) \in \mathbb{N}$ then c is a channel with finite capacity
  - if *cap*$(c) = \infty$ then c has an infinite capacity
  - if *cap*$(c) > 0$, there is some "delay" between sending and receipt
  - if *cap*$(c) = 0$, then communication via c amounts to handshaking

Channels

- Process P_i = *program graph* PG_i + *communication actions*

  $c!v$ transmit the value v along channel c

  $c?x$ receive a message via channel c and assign it to variable x

- $\mathit{Comm} = \{ c!v, \; c?x \mid c \in \mathit{Chan}, \; v \in \mathit{dom}(c), \; x \in \mathit{Var}. \; \mathit{dom}(x) \supseteq \mathit{dom}(c) \}$

- Sending and receiving a message
  - $c!v$ puts the value v at the rear of the buffer c (if c is not full)
  - $c?x$ retrieves the front element of the buffer and assigns it to x (if c is not empty)
  - if $\mathit{cap}(c) = 0$, channel c has *no* buffer
  - if $\mathit{cap}(c) = 0$, sending and receiving can takes place simultaneously this is called synchronous message passing or handshaking
  - if $\mathit{cap}(c) > 0$, sending and receiving can never take place simultaneously this is called *asynchronous message passing*

Channel systems

A program graph over $(\mathit{Var}, \mathit{Chan})$ is a tuple

$$\mathit{PG} = (\mathit{Loc}, \mathit{Act}, \mathit{Effect}, \rightarrow, \mathit{Loc}_0, g_0)$$

where

$$\rightarrow \quad \subseteq \quad \mathit{Loc} \times (\mathit{Cond}(\mathit{Var}) \times \mathit{Act}) \times \mathit{Loc} \;\; \cup \;\; \underbrace{\mathit{Loc} \times \mathit{Comm} \times \mathit{Loc}}_{\text{communication actions}}$$

A *channel system* CS over $(\bigcup_{0<i\leqslant n} \mathit{Var}_i, \mathit{Chan})$:

$$\mathit{CS} \;=\; [\mathit{PG}_1 \mid \ldots \mid \mathit{PG}_n]$$

with program graphs PG_i over $(\mathit{Var}_i, \mathit{Chan})$

Communication actions

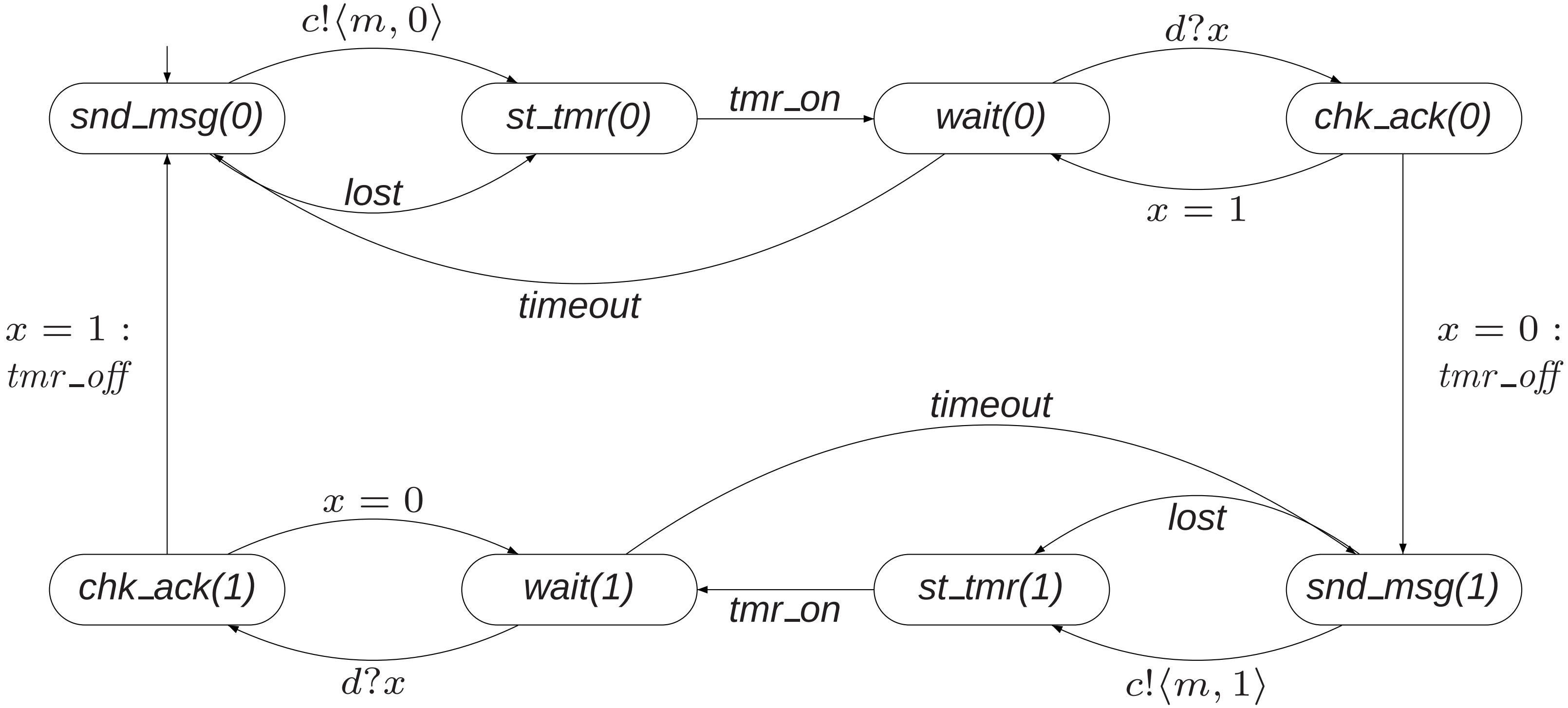
- *Handshaking*
  - if *cap*$(c) = 0$, then process P_i can perform $\ell_i \xrightarrow{c!v} \ell_i'$ only
  - ... if P_j, say, can perform $\ell_j \xrightarrow{c?x} \ell_j'$
  - the effect corresponds to the (atomic) *distributed* assignment $x := v$.

- *Asynchronous message passing*
  - if *cap*$(c) > 0$, then process P_i can perform $\ell_i \xrightarrow{c!v} \ell_i'$
  - ... if and only if less than *cap*(c) messages are stored in c
  - P_j may perform $\ell_j \xrightarrow{c?v} \ell_j'$ if and only if the buffer of c is not empty
  - then the first element v of the buffer is extracted and assigned to x (atomically)

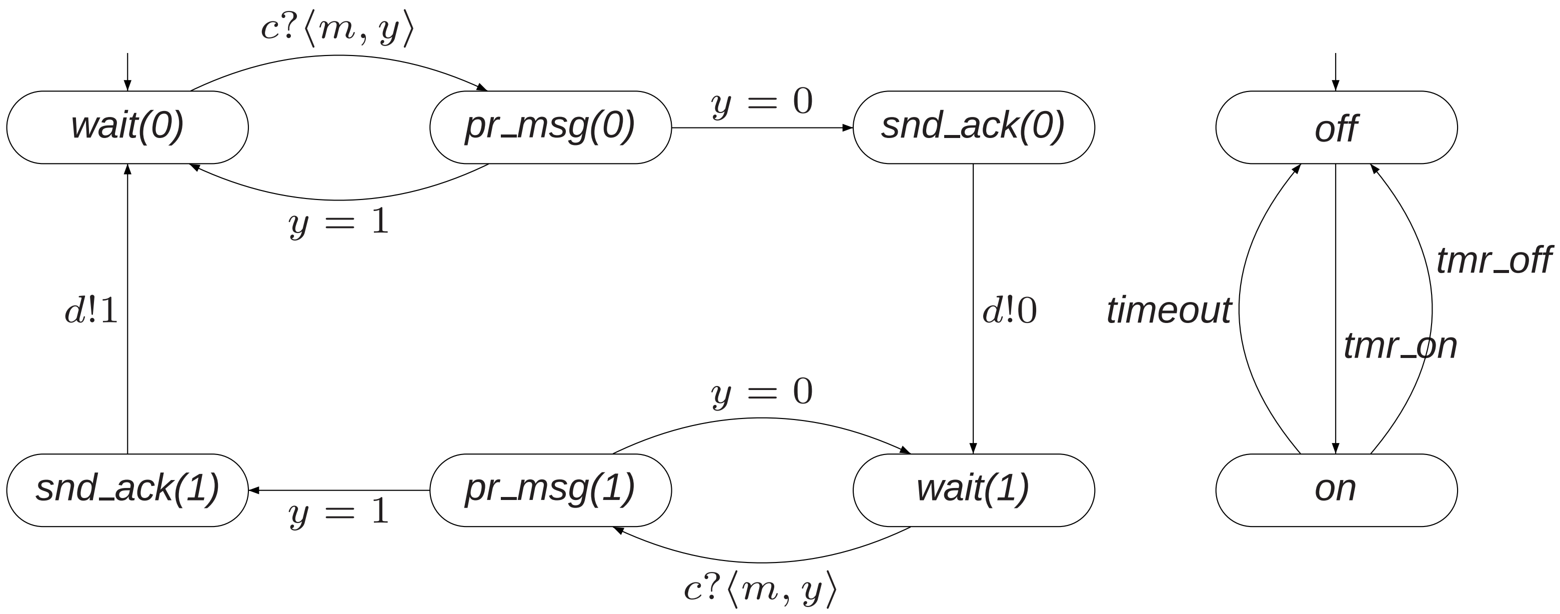
	executable if ...	effect
$c!v$	c is not "full"	$Enqueue(c, v)$
$c?x$	c is not empty	$\langle x := Front(c)\ ;\ Dequeue(c) \rangle$;

The alternating bit protocol

The alternating bit protocol: sender



The alternating bit protocol: receiver



Channel evaluations

- A *channel evaluation* ξ is
  - a mapping from channel $c \in \mathit{Chan}$ onto a sequence $\xi(c) \in \mathit{dom}(c)^*$ such that
  - current length cannot exceed the capacity of c: $\mathit{len}(\xi(c)) \leqslant \mathit{cap}(c)$
  - $\xi(c) = v_1\, v_2\, \ldots\, v_k$ ($\mathit{cap}(c) \geqslant k$) denotes v_1 is at front of buffer etc.

- $\xi[c := v_1 \ldots v_k]$ denotes the channel evaluation

$$\xi[c := v_1 \ldots v_k](c') \;=\; \begin{cases} \xi(c') & \text{if } c \neq c' \\ v_1 \ldots v_k & \text{if } c = c'. \end{cases}$$

- Initial channel evaluation ξ_0 equals $\xi_0(c) = \varepsilon$ for any c

Transition system semantics of a channel system

Let $\mathit{CS} = [PG_1 \mid \ldots \mid PG_n]$ be a *channel system* over $(\mathit{Chan}, \mathit{Var})$ with

$$\mathit{PG}_i = (\mathit{Loc}_i, \mathit{Act}_i, \mathit{Effect}_i, \rightsquigarrow_i, \mathit{Loc}_{0,i}, g_{0,i}), \qquad \text{for } 0 < i \leqslant n$$

$\mathit{TS}(\mathit{CS})$ is the *transition system* $(S, \mathit{Act}, \rightarrow, I, \mathit{AP}, L)$ where:

- $S \;=\; (\mathit{Loc}_1 \times \ldots \times \mathit{Loc}_n) \times \mathit{Eval}(\mathit{Var}) \times \mathit{Eval}(\mathit{Chan})$
- $\mathit{Act} \;=\; \left(\biguplus_{0<i\leqslant n} \mathit{Act}_i\right) \uplus \{\, \tau \,\}$
- $\rightarrow$ is defined by the inference rules on the next slides
- $I = \left\{ \langle \ell_1, \ldots, \ell_n, \eta, \xi_0 \rangle \;\mid\; \forall i.\ (\ell_i \in Loc_{0,i} \wedge \eta \models g_{0,i}) \wedge \forall c.\ \xi_0(c) = \varepsilon \right\}$
- $\mathit{AP} \;=\; \biguplus_{0<i\leqslant n} \mathit{Loc}_i \;\uplus\; \mathit{Cond}(\mathit{Var})$
- $L(\langle \ell_1, \ldots, \ell_n, \eta, \xi \rangle) \;=\; \{\, \ell_1, \ldots, \ell_n \,\} \,\cup\, \{\, g \in \mathit{Cond}(\mathit{Var}) \mid \eta \models g \,\}$

Inference rules (I)

- Interleaving for $\alpha \in \mathit{Act}_i$:

$$\frac{\ell_i \xrightarrow{g:\alpha} \ell_i' \quad \wedge \quad \eta \models g}{\langle \ell_1, \ldots, \ell_i, \ldots, \ell_n, \eta, \xi \rangle \xrightarrow{\alpha} \langle \ell_1, \ldots, \ell_i', \ldots, \ell_n, \eta', \xi \rangle}$$

where $\eta' = \mathit{Effect}(\alpha, \eta)$

- Synchronous message passing over $c \in \mathit{Chan}$, $\mathit{cap}(c) = 0$:

$$\frac{\ell_i \xrightarrow{c?x} \ell_i' \quad \wedge \quad \ell_j \xrightarrow{c!v} \ell_j' \quad \wedge \quad i \neq j}{\langle \ell_1, \ldots, \ell_i, \ldots, \ell_j, \ldots, \ell_n, \eta, \xi \rangle \xrightarrow{\tau} \langle \ell_1, \ldots, \ell_i', \ldots, \ell_j', \ldots, \ell_n, \eta', \xi \rangle}$$

where $\eta' = \eta[x := v]$.

Inference rules (II)

- Asynchronous message passing for $c \in \mathit{Chan}$, $\mathit{cap}(c) > 0$:
  - receive a value along channel c and assign it to variable x:

$$\frac{\ell_i \xrightarrow{c?x} \ell_i' \quad \wedge \quad \mathit{len}(\xi(c)) = k > 0 \quad \wedge \quad \xi(c) \; = \; v_1 \ldots v_k}{\langle \ell_1, \ldots, \ell_i, \ldots, \ell_n, \eta, \xi \rangle \xrightarrow{\tau} \langle \ell_1, \ldots, \ell_i', \ldots, \ell_n, \eta', \xi' \rangle}$$

where $\eta' = \eta[x := v_1]$ and $\xi' = \xi[c := v_2 \ldots v_k]$.

  - transmit value $v \in \mathit{dom}(c)$ over channel c:

$$\frac{\ell_i \xrightarrow{c!v} \ell_i' \quad \wedge \quad \mathit{len}(\xi(c)) = k < \mathit{cap}(c) \quad \wedge \quad \xi(c) \; = \; v_1 \ldots v_k}{\langle \ell_1, \ldots, \ell_i, \ldots, \ell_n, \eta, \xi \rangle \xrightarrow{\tau} \langle \ell_1, \ldots, \ell_i', \ldots, \ell_n, \eta, \xi' \rangle}$$

where $\xi' = \xi[c := v_1 \, v_2 \ldots v_k \, v]$.

Handling unexpected messages

sender S	timer	receiver R	channel c	channel d	event
$snd\_msg(0)$	off	$wait(0)$	$\varnothing$	$\varnothing$	
$st\_tmr(0)$	off	$wait(0)$	$\langle m, 0\rangle$	$\varnothing$	message with bit 0 transmitted
$wait(0)$	on	$wait(0)$	$\langle m, 0\rangle$	$\varnothing$	
$snd\_msg(0)$	off	$wait(0)$	$\langle m, 0\rangle$	$\varnothing$	timeout
$st\_tmr(0)$	off	$wait(0)$	$\langle m, 0\rangle\ \langle m, 0\rangle$	$\varnothing$	retransmission
$st\_tmr(0)$	off	$pr\_msg(0)$	$\langle m, 0\rangle$	$\varnothing$	receiver reads first message
$st\_tmr(0)$	off	snd_ack(0)	$\langle m, 0\rangle$	$\varnothing$	
$st\_tmr(0)$	off	$wait(1)$	$\langle m, 0\rangle$	0	receiver changes into mode-1
$st\_tmr(0)$	off	$pr\_msg(1)$	$\varnothing$	0	receiver reads retransmission and ignores it
$st\_tmr(0)$	off	$wait(1)$	$\varnothing$	0	
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	

nanoPromela

- Promela (Process Meta Language) is modeling language for SPIN
  - most widely used model checker SPIN
  - developed by Gerard Holzmann (Bell Labs, NASA JPL)
  - ACM Software Award 2002
- nanoPromela is the core of Promela
  - shared variables and channel-based communication
  - formal semantics of a Promela model is a channel system
  - processes are defined by means of a guarded command language
- No actions, statements describe effect of actions

nanoPromela

nanoPromela-program $\overline{\mathcal{P}} = [\mathcal{P}_1 | \ldots | \mathcal{P}_n]$ with $\mathcal{P}_i$ processes

A process is specified by a statement:

$$
\begin{array}{lll}
\text{stmt} & ::= & \texttt{skip} \;\big|\; x := \text{expr} \;\big|\; c?x \;\big|\; c!\text{expr} \;\big| \\
 & & \text{stmt}_1 \,;\, \text{stmt}_2 \;\big|\; \texttt{atomic}\{\text{assignments}\} \;\big| \\
 & & \textbf{if} \quad :: g_1 \Rightarrow \text{stmt}_1 \quad \ldots \quad :: g_n \Rightarrow \text{stmt}_n \quad \textbf{fi} \quad \big| \\
 & & \textbf{do} \quad :: g_1 \Rightarrow \text{stmt}_1 \quad \ldots \quad :: g_n \Rightarrow \text{stmt}_n \quad \textbf{do} \\
\text{assignments} & ::= & x_1 := \text{expr}_1 \,;\, x_2 := \text{expr}_2 \,;\, \ldots \,;\, x_m := \text{expr}_m
\end{array}
$$

x is a variable in *Var*, expr an expression and c a channel, g_i a guard

assume the Promela specification is type-consistent

Conditional statements

$$\textbf{if} \;\; :: g_1 \Rightarrow \text{stmt}_1 \; \ldots \; :: g_n \Rightarrow \text{stmt}_n \; \textbf{fi}$$

- Nondeterministic choice between statements stmt$_i$ for which g_i holds
- Test-and-set semantics: (deviation from Promela)
  - guard evaluation + selection of enabled command + execution first atomic step of selected statement is all performed atomically
- The **if**–**fi**–command blocks if no guard holds
  - parallel processes may unblock a process by changing shared variables
  - e.g., when $y=0$, **if** $:: y > 0 \Rightarrow x := 42$ **fi** waits until y exceeds 0
- Standard abbreviations:
  - **if** g **then** stmt$_1$ **else** stmt$_2$ **fi** $\equiv$ **if** $:: g \Rightarrow$ stmt$_1$ $:: \neg g \Rightarrow$ stmt$_2$ **fi**
  - **if** g **then** stmt$_1$ **fi** $\equiv$ **if** $:: g \Rightarrow$ stmt$_1$ $:: \neg g \Rightarrow$ `skip` **fi**

Iteration statements

$$\textbf{do} \ :: g_1 \Rightarrow \text{stmt}_1 \ \ldots \ :: g_n \Rightarrow \text{stmt}_n \ \textbf{od}$$

- Iterative execution of nondeterministic choice among $g_i \Rightarrow \text{stmt}_i$
  - where guard g_i holds in the current state
- No blocking if all guards are violated; instead, loop is aborted
- $\textbf{do} \ :: g \Rightarrow \text{stmt} \ \textbf{od} \equiv \textbf{while} \ g \ \textbf{do} \ \text{stmt} \ \textbf{od}$
- No break-statements to abort a loop (deviation from Promela)

Peterson's algorithm

The nanoPromela-code of process $\mathcal{P}_1$ is given by the statement:

$$
\begin{array}{lll}
\textbf{do} & :: \ \text{true} \Rightarrow & \texttt{skip}; \\
& & \texttt{atomic}\{b_1 := \text{true}; x := 2\}; \\
& & \textbf{if} \ :: \ (x = 1) \vee \neg b_2 \ \Rightarrow \ \mathit{crit}_1 := \text{true} \ \textbf{fi} \\
& & \texttt{atomic}\{\mathit{crit}_1 := \text{false}; b_1 := \text{false}\} \\
\textbf{od} & &
\end{array}
$$

Beverage vending machine

The following nanoPromela program describes its behaviour:

$$
\begin{array}{lll}
\textbf{do} & :: & \text{true} \Rightarrow \\
& & \quad \texttt{skip}; \\
& & \quad \textbf{if} \quad :: \ \textit{nsprite} > 0 \ \Rightarrow \ \textit{nsprite} := \textit{nsprite} - 1 \\
& & \quad \phantom{\textbf{if}} \quad :: \ \textit{nbeer} > 0 \ \Rightarrow \ \textit{nbeer} := \textit{nbeer} - 1 \\
& & \quad \phantom{\textbf{if}} \quad :: \ \textit{nsprite} = \textit{nbeer} = 0 \Rightarrow \texttt{skip} \\
& & \quad \textbf{fi} \\
& :: & \text{true} \Rightarrow \ \texttt{atomic}\{\textit{nbeer} := \textit{max}; \textit{nsprite} := \textit{max}\} \\
\textbf{od} & &
\end{array}
$$

Formal semantics

The *semantics* of a nanoPromela-statement over $(\textit{Var}, \textit{Chan})$ is a *program graph* over $(\textit{Var}, \textit{Chan})$.

The program graphs $\textit{PG}_1, \ldots, \textit{PG}_n$ for the processes $\mathcal{P}_1, \ldots, \mathcal{P}_n$ of a nanoPromela-program $\overline{\mathcal{P}} = [\mathcal{P}_1 | \ldots | \mathcal{P}_n]$ constitute a *channel system* over $(\textit{Var}, \textit{Chan})$

Example:

$$\begin{array}{lclcccl} \text{loop} & = & \textbf{do} & :: & x > 1 & \Rightarrow & y := x + y \\ & & & :: & y < x & \Rightarrow & x := 0;\ y := x \\ & & \textbf{od} & & & & \end{array}$$

Sub-statements

Inference rules

$$\frac{}{\texttt{skip} \xrightarrow{\text{true: } \textit{id}} \text{exit}}$$

where *id* denotes an action that does not change the values of the variables

$$\frac{}{x := \text{expr} \xrightarrow{\text{true : assign}(x, \text{expr})} \text{exit}}$$

assign$(x$, expr$)$ denotes the action that only changes x, no other variables

$$\frac{}{c?x \xrightarrow{c?x} \text{exit}} \qquad \frac{}{c!\text{expr} \xrightarrow{c!\text{expr}} \text{exit}}$$

Inference rules

$$\frac{}{\texttt{atomic}\{x_1 := \text{expr}_1; \ldots ; x_m := \text{expr}_m\} \xrightarrow{\text{true} : \alpha_m} \text{exit}}$$

where $\alpha_0 = \mathit{id}$, $\alpha_i = \mathit{Effect}(\text{assign}(x_i, \text{expr}_i), \mathit{Effect}(\alpha_{i-1}, \eta))$ for $1 \leqslant i \leqslant m$

$$\frac{\text{stmt}_1 \xrightarrow{g:\alpha} \text{stmt}_1' \neq \text{exit}}{\text{stmt}_1; \text{stmt}_2 \xrightarrow{g:\alpha} \text{stmt}_1'; \text{stmt}_2}$$

$$\frac{\text{stmt}_1 \xrightarrow{g:\alpha} \text{exit}}{\text{stmt}_1; \text{stmt}_2 \xrightarrow{g:\alpha} \text{stmt}_2}$$

Inference rules

$$\frac{\text{stmt}_i \xrightarrow{h:\alpha} \text{stmt}_i'}{\text{cond\_cmd} \xrightarrow{g_i \wedge h:\alpha} \text{stmt}_i'}$$

$$\frac{\text{stmt}_i \xrightarrow{h:\alpha} \text{stmt}_i' \neq \text{exit}}{\text{loop} \xrightarrow{g_i \wedge h:\alpha} \text{stmt}_i'; \text{loop}} \qquad \frac{\text{stmt}_i \xrightarrow{h:\alpha} \text{exit}}{\text{loop} \xrightarrow{g_i \wedge h:\alpha} \text{loop}}$$

$$\frac{}{\text{loop} \xrightarrow{\neg g_1 \wedge \ldots \wedge \neg g_n} \text{exit}}$$

Overview Lecture #4

- *Concurrency*
  - The interleaving paradigm
- Communication principles
  - Shared variable “communication”
  - Handshaking
  - Synchronous communication
- Channel systems

$\Rightarrow$ The state-space explosion problem

Sequential programs

- The $\#$ states of a simple program graph is:

$$\left| \#\text{program locations} \right| \cdot \prod_{variable\ x} |\, dom(x) \,|$$

$\Rightarrow$ number of states grows *exponentially* in the number of program variables

- N variables with k possible values each yields k^N states
- this is called *the state-space explosion problem*

- A program with 10 locations, 3 bools, 5 integers (in range $0 \ldots 9$):

$$10 \cdot 2^3 \cdot 10^5 = 800,000 \text{ states}$$

- Adding a single 50-positions bit-array yields $800,000 \cdot 2^{50}$ states

Concurrent programs

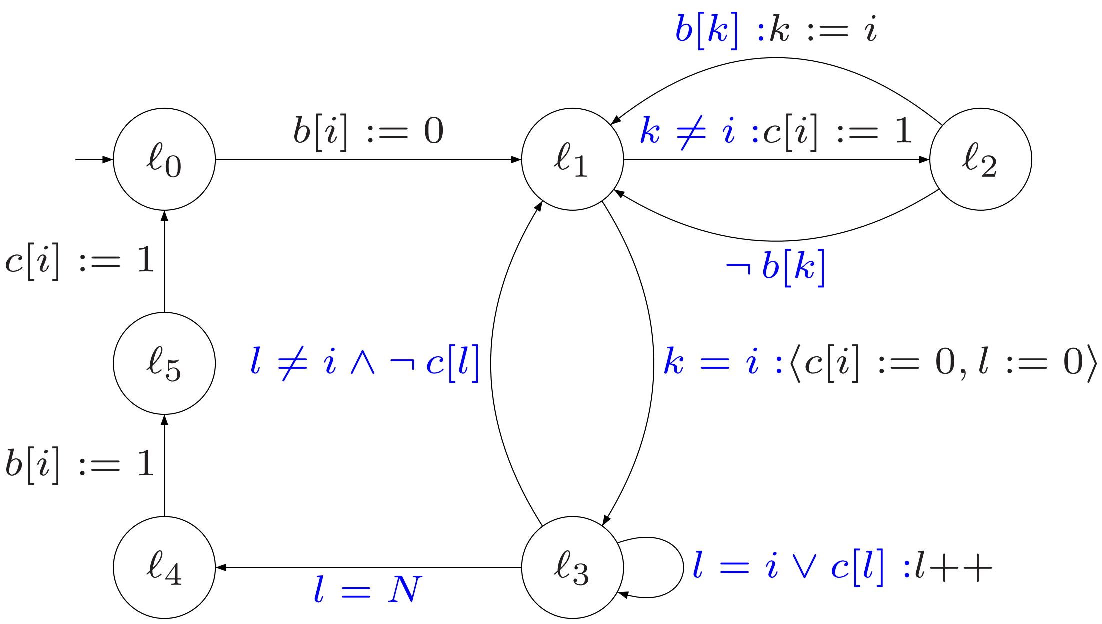
- The $\#$ states of $P \equiv P_1 \parallel \ldots \parallel P_n$ is maximally:

$$\#\text{states of } P_1 \times \ldots \times \#\text{states of } P_n$$

$\Rightarrow$ $\#$ states grows *exponentially* with the number of components

- The composition of N components of size k each yields k^N states
- This is called *the state-space explosion problem*

Dijkstra's mutual exclusion program



- two bit-arrays of size N
- global variable k
  - with value in $1, \ldots, N$
- local variable l
  - with value in $1, \ldots, N$
- 6 program locations per process

$\Rightarrow$ totally $2^{2N} \cdot N \cdot (6N)^N$ states

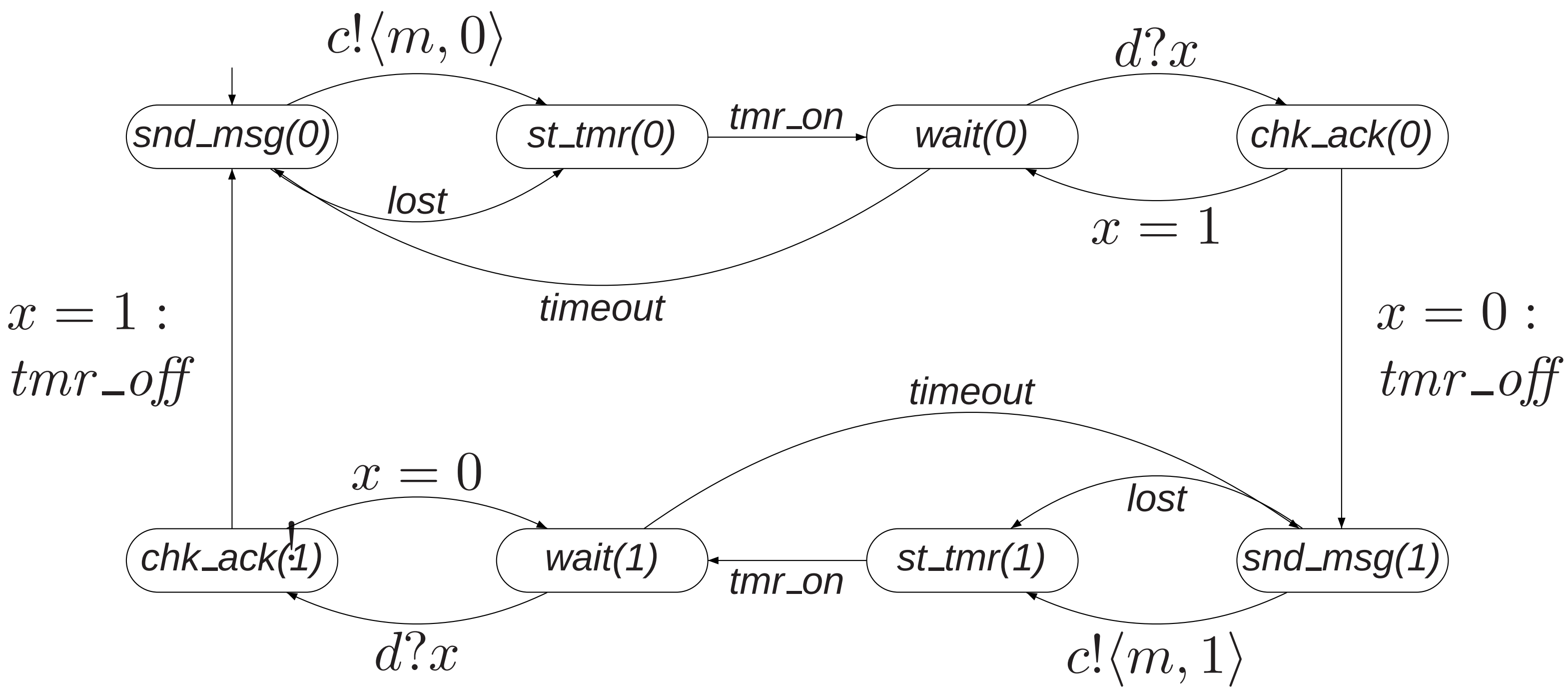
Channel systems

- Asynchronous communication of processes via *channels*
  - each channel c has a bounded capacity $cap(c)$
  - if a channel has capacity 0, we obtain handshaking
- $\#$ states of system with N components and K channels is:

$$\prod_{i=1}^{N} \left(|\#\text{program locations}| \prod_{variable\ x} |\, dom(x)| \right) \cdot \prod_{j=1}^{K} |dom(c_j)|^{cap(c_j)}$$

this is the underlying structure of Promela

The alternating bit protocol



channel capacity 10, and datums are bits, yields $2{\cdot}8{\cdot}6{\cdot}4^{10}{\cdot}2^{10} = 3{\cdot}2^{35} \approx 10^{11}$ states

Summary of Chapter 2

- Transition systems are fundamental for modeling software and hardware
- Interleaving = execution of independent concurrent processes by nondeterminism
- For shared variable communication use composition on program graphs
- Handshaking on a set H of actions amounts to
  - executing action $\notin H$ autonomously (= interleaving)
  - those in H simultaneously
- Channel systems = program graphs + first-in first-out communication channels
  - handshaking for channels of capacity 0
  - asynchronous message passing when capacity exceeds 0
  - semantical model of Promela
- Size of transition systems grows exponentially
  - in the number of concurrent components and the number of variables