RHEINISCH-WESTFÄLISCHE TECHNISCHE HOCHSCHULE AACHEN

LEHRSTUHL FÜR INFORMATIK 2
RWTH Aachen · D-52056 Aachen
http://www-i2.informatik.rwth-aachen.de/

Prof. Dr. Ir. J.-P. Katoen
M. Neuhäußer

Introduction to Model Checking
Winter term 08/09

– Series 4 –

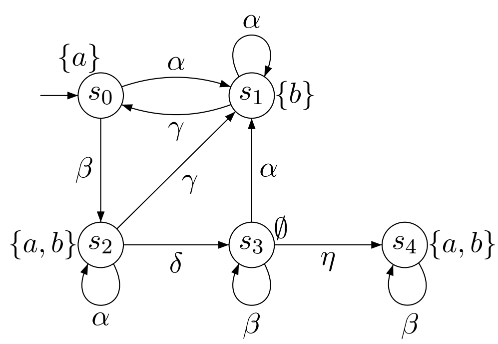
Hand in on November 21 before the exercise class.

Exercise 1 (3 points)

Let P denote the set of traces of the form $\sigma = A_0 A_1 A_2 \cdots \in \left(2^{AP}\right)^\omega$ such that

$$\overset{\infty}{\exists}\, k.\; A_k = \{a, b\} \quad \wedge \quad \exists n \geq 0.\; \forall k > n.\; \left(a \in A_k \Rightarrow b \in A_{k+1}\right).$$

Consider the following fairness assumptions with respect to the transition system TS outlined on the right:



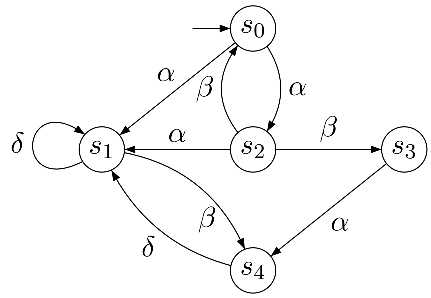
a) $\mathcal{F}_1 = \left(\{\{\alpha\}\}, \{\{\beta\}, \{\delta, \gamma\}, \{\eta\}\}, \emptyset\right)$.
   Decide whether $TS \models_{\mathcal{F}_1} P$.

b) $\mathcal{F}_2 = \Big(\{\{\alpha\}\}, \{\{\beta\}, \{\gamma\}\}, \{\{\eta\}\}\Big)$.
   Decide whether $TS \models_{\mathcal{F}_2} P$.

Justify your answers!

Exercise 2 (3 points)

Consider the transition system TS on the right (where atomic propositions are omitted). Decide which of the following fairness assumptions $\mathcal{F}_i$ are realizable for TS. Justify your answers!



a) $\mathcal{F}_1 = \left(\{\{\alpha\}\}, \{\{\delta\}\}, \{\{\alpha, \beta\}\}\right)$

b) $\mathcal{F}_2 = \left(\{\{\alpha, \delta\}\}, \{\{\alpha, \beta\}\}, \{\{\delta\}\}\right)$

c) $\mathcal{F}_3 = \left(\{\{\alpha, \delta\}, \{\beta\}\}, \{\{\alpha, \beta\}\}, \{\{\delta\}\}\right)$

Exercise 3 (1 + 1 points)

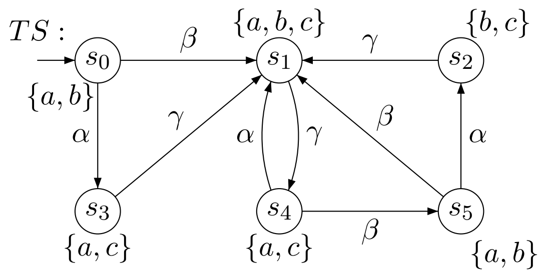
Let $n \geq 1$. Consider the language $L_n \subseteq \Sigma^*$ over the alphabet $\Sigma = \{A, B\}$ that consists of all finite words where the symbol B is on position n from the right, i.e., L_n contains exactly the words $A_1 A_2 \ldots A_k \in \{A, B\}^*$ where $k \geq n$ and $A_{k-n+1} = B$. For instance, the word $ABBAABAB$ is in L_3.

a) Construct an NFA $\mathcal{A}_n$ with at most $n + 1$ states such that $\mathcal{L}(\mathcal{A}_n) = L_n$.

b) Determinize this NFA $\mathcal{A}_n$ using the powerset construction algorithm.

Exercise 4 **(2 + 2 points)**

Consider the following transition system TS



and the regular safety property

$$P_{safe} = \begin{array}{c}\text{"always if } a \text{ is valid and } b \wedge \neg c \text{ was valid somewhere before,} \\ \text{then } a \text{ and } b \text{ do not hold thereafter at least until } c \text{ holds"}\end{array}$$

As an example, it holds:

$$\begin{aligned} \{b\}\emptyset\{a,b\}\{a,b,c\} &\in pref(P_{safe}) \\ \{a,b\}\{a,b\}\emptyset\{b,c\} &\in pref(P_{safe}) \\ \{b\}\{a,c\}\{a\}\{a,b,c\} &\in BadPref(P_{safe}) \\ \{b\}\{a,c\}\{a,c\}\{a\} &\in BadPref(P_{safe}) \end{aligned}$$

Questions:

a) Define an NFA $\mathcal{A}$ such that $\mathcal{L}(\mathcal{A}) = MinBadPref(P_{safe})$.

b) Decide whether $TS \models P_{safe}$ using the $TS \otimes \mathcal{A}$ construction.
Provide a counterexample if $TS \not\models P_{safe}$.