RHEINISCH-WESTFÄLISCHE TECHNISCHE HOCHSCHULE AACHEN

LEHRSTUHL FÜR INFORMATIK 2
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Introduction to Model Checking
Winter term 08/09

– Series 6 –

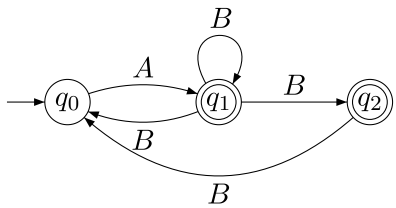
Hand in on **December 12** before the exercise class.

Exercise 1 (2 points)

Show that the class of languages accepted by DBA is not closed under complement!

Exercise 2 (2 points)

Consider the GNBA outlined on the right with acceptance sets $F_1 = \{q_1\}$ and $F_2 = \{q_2\}$. Construct an equivalent NBA using the transformation introduced in the lecture.

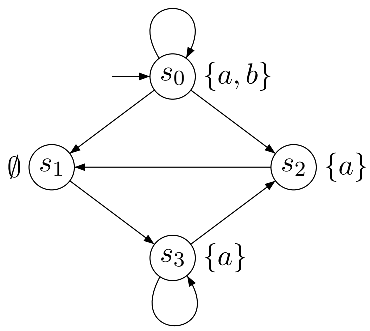


Exercise 3 (1 + 2 + 2 + 1 points)

We consider model checking of the ω–regular properties P_1 and P_2 which are defined over the set $AP = \{a, b\}$ of atomic propositions:

$$P_1 := \left\{\sigma \in \left(2^{AP}\right)^\omega \,\middle|\, \sigma \models \Box\Diamond a \rightarrow \Box\Diamond b\right\} \qquad \text{and}$$
$$P_2 := \left\{\sigma \in \left(2^{AP}\right)^\omega \,\middle|\, \sigma \models \Diamond(a \wedge \bigcirc a)\right\}.$$

Further, our model is represented by the transition system TS which is given on the right. We want to check whether $TS \models P_i$ for $i = 1, 2$ using the nested depth–first search algorithm from the lecture. Therefore proceed as follows:



a) Derive a NBA $\mathcal{A}_i$ for the complement $\overline{P}_i$ of P_i for $i = 1, 2$.
More precisely, for $\mathcal{A}_i$ it must hold $\mathcal{L}_\omega(\mathcal{A}_i) = \overline{P}_i$.
Hint: Four, respectively three states suffice. Derive the automata directly.

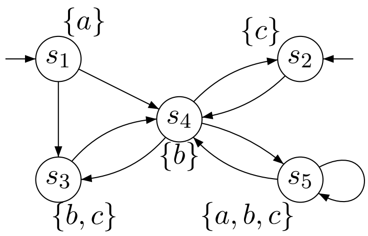
b) Outline the reachable fragment of the product transition system $TS \otimes \mathcal{A}_i$.

c) Sketch the main steps of the nested depth–first search algorithm for the persistency check on $TS \otimes \mathcal{A}_i$.

d) Provide the counterexample computed by the algorithm in case that $TS \not\models P_i$.

Exercise 4 (3 points)

Consider the transition system TS over the set of atomic propositions $AP = \{a, b, c\}$:



Decide for each of the LTL formulas φ_i below, whether $TS \models \varphi_i$ holds. Justify your answers!
If $TS \not\models \varphi_i$, provide a path $\pi \in Paths(TS)$ such that $\pi \not\models \varphi_i$.

$$\varphi_1 = \Diamond\Box c \qquad \varphi_4 = \Box a$$
$$\varphi_2 = \Box\Diamond c \qquad \varphi_5 = a \mathsf{U} \Box(b \vee c)$$
$$\varphi_3 = \bigcirc\neg c \rightarrow \bigcirc\bigcirc c \qquad \varphi_6 = (\bigcirc\bigcirc\, b) \mathsf{U} (b \vee c)$$

Exercise 5 (6 points)

Prove or disprove the following equivalences of LTL–formulas:

$$\Box\varphi \rightarrow \Diamond\psi \equiv \varphi \mathsf{U} (\psi \vee \neg\varphi) \qquad \Diamond\Box\varphi \rightarrow \Box\Diamond\psi \equiv \Box(\varphi \mathsf{U} (\psi \vee \neg\varphi))$$
$$\Box\Diamond\varphi \rightarrow \Box\Diamond\psi \equiv \Box(\varphi \rightarrow \Diamond\psi) \qquad \Diamond\,(\varphi \mathsf{U} \psi) \equiv \Diamond\psi$$