RHEINISCH-WESTFÄLISCHE TECHNISCHE HOCHSCHULE AACHEN

LEHRSTUHL FÜR INFORMATIK 2
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Introduction to Model Checking
Winter term 08/09

– Series 8 –

Hand in on January 09 before the exercise class.

Exercise 1 (2 + 2 points)

We consider the release operator R which is defined as $\varphi \mathsf{R} \psi := \neg(\neg\varphi \mathsf{U} \neg\psi)$.

a) Informally describe the meaning of the expansion law for the release operator R. Then prove its correctness formally.

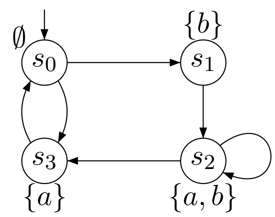
b) Prove the following two equivalence laws that express R by W and vice versa:

- $\varphi \mathsf{R} \psi \equiv (\neg\varphi \wedge \psi)\, \mathsf{W}\, (\varphi \wedge \psi)$
- $\varphi \mathsf{W} \psi \equiv (\neg\varphi \vee \psi)\, \mathsf{R}\, (\varphi \vee \psi)$

Exercise 2 (0.5 + 1.5 + 2 + 1 + 1 + 2 points)

We consider the LTL formula $\varphi = \Box(a \rightarrow (\neg b \mathsf{U} (a \wedge b)))$ over the set $AP = \{a, b\}$ of atomic propositions and want to check $TS \models \varphi$ wrt. the transition system outlined on the right.

TS :



a) To check $TS \models \varphi$, convert $\neg\varphi$ into an equivalent LTL-formula ψ which is constructed according to the following grammar:

$$\varphi ::= \mathit{true} \mid \mathit{false} \mid a \mid b \mid \varphi \wedge \varphi \mid \neg\varphi \mid \bigcirc\varphi \mid \varphi \mathsf{U} \varphi.$$

Then construct $\mathit{closure}(\psi)$.

b) Give the elementary sets wrt. $\mathit{closure}(\psi)$!

c) Construct the GNBA $\mathcal{G}_\psi$ by providing its initial states, its acceptance set and its transition relation. Use the algorithm given in the lecture.
Hint: It suffices to provide the transition relation as a table.

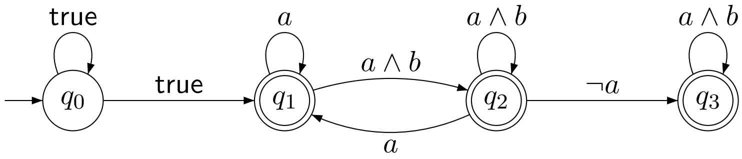
d) Now, construct an NBA $\mathcal{A}_{\neg\varphi}$ **directly** from $\neg\varphi$, i.e. without relying on $\mathcal{G}_\psi$.
Hint: Four states suffice!

e) Construct $TS \otimes \mathcal{A}_{\neg\varphi}$.

f) Use the Nested DFS algorithm from the lecture to check $TS \models \varphi$. Therefore sketch the algorithm's main steps and interpret its outcome!

Exercise 3 **(2 points)**

Consider the GNBA $\mathcal{G}$ over the alphabet $\Sigma = 2^{\{a,b\}}$ and the set $\mathcal{F} = \{\{q_1, q_3\}, \{q_2\}\}$ of acceptance sets:



a) Provide an LTL formula φ such that $Word(\varphi) = \mathcal{L}_\omega(\mathcal{G})$. Justify your answer!

b) Depict an NBA $\mathcal{A}$ with $\mathcal{L}_\omega(\mathcal{A}) = \mathcal{L}_\omega(\mathcal{G})$.

Merry Christmas and a happy new year!!!