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LEHRSTUHL FÜR INFORMATIK 2
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Introduction to Model Checking
Winter term 08/09

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– Series 10 –

Hand in on January 23 before the exercise class.

Exercise 1 (3 + 1 points)

Consider the fragment ECTL of CTL which consists of formulas built according to the following grammar:

$$\Phi ::= a \mid \neg a \mid \Phi \wedge \Phi \mid \exists \varphi$$
$$\varphi ::= \bigcirc \Phi \mid \Box \Phi \mid \Phi \mathsf{U} \Phi$$

Therefore, ECTL-formulas are built by atomic propositions, negated atomic propositions, the boolean connective $\wedge$ and the path quantifier $\exists$ together with the modalities $\bigcirc$, $\Box$ and U.
For two transition systems $TS_1 = (S_1, Act, \rightarrow_1, I_1, AP, L_1)$ and $TS_2 = (S_2, Act, \rightarrow_2, I_2, AP, L_2)$, we define $TS_1 \subseteq TS_2$ iff $S_1 \subseteq S_2$, $\rightarrow_1 \subseteq \rightarrow_2$, $I_1 = I_2$ and $L_1(s) = L_2(s)$ for all $s \in S_1$.

(a) Prove, that for all ECTL-formulas Φ and all transition systems TS_1, TS_2 with $TS_1 \subseteq TS_2$, it holds:

$$TS_1 \models \Phi \quad \Longrightarrow \quad TS_2 \models \Phi.$$

(b) Give a CTL-formula which is not equivalent to any other ECTL-formula. Justify your answer!

Exercise 2 (1 points)

Transform the CTL-formula $\Phi = \neg\forall\Diamond\big(\forall\,(\forall\Box b)\,\mathsf{U}\,(\forall \bigcirc a)\big)$ into an equivalent CTL-formula in existential normal form.

Exercise 3 (2 + 2 points)

We consider again the incomparable expressiveness of CTL and LTL.

(a) Using Theorem 6.18, prove that there does not exist an equivalent LTL-formula for the CTL-formula $\Phi_1 = \forall\Diamond(a \wedge \exists \bigcirc a)$.

(b) Now prove directly (i.e. without Theorem 6.18), that there does not exist an equivalent LTL-formula for the CTL-formula $\Phi_2 = \forall\Diamond\exists \bigcirc \forall\Diamond\neg a$.
Hint: Argument by contraposition!

Exercise 4 (2 + 3 points)

Consider the following CTL-formulas

$$\Phi_1 = \exists\Diamond\forall\Box c \quad \text{and} \quad \Phi_2 = \forall(a\mathsf{U}\forall\Diamond c)$$

and the transition system outlined on the right. Decide whether $TS \models \Phi_i$ for $i = 1, 2$ using the CTL model checking algorithm from the lecture. Sketch its main steps!