

– Series 5 –

Hand in on June 2 before the exercise class.

Exercise 1

(3 points)

Let  $AP = \{a, b, c\}$ . Consider the following linear time properties:

- (a) If  $a$  becomes valid, afterwards  $b$  stays valid ad infinitum or until  $c$  holds.
- (b) Between two neighbouring occurrences of  $a$ ,  $b$  always holds.
- (c) Between two neighbouring occurrences of  $a$ ,  $b$  occurs more often than  $c$ .
- (d)  $a \wedge \neg b$  and  $b \wedge \neg a$  are valid in alternation or until  $c$  becomes valid.

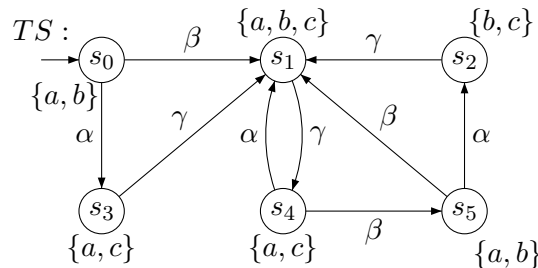
For each property  $P_i$  ( $1 \leq i \leq 4$ ), decide if it is a regular safety property (argument why!) and if so, define the NFA  $\mathcal{A}_i$  with  $\mathcal{L}(\mathcal{A}_i) = \text{BadPref}(P_i)$ .

*Hint: You may use propositional formulas over the set  $AP$  as transition labels.*

Exercise 2

(2 + 2 points)

Consider the following transition system  $TS$



and the regular safety property

$P_{safe} =$  “always if  $a$  is valid and  $b \wedge \neg c$  was valid somewhere before, then  $a$  and  $b$  do not hold thereafter at least until  $c$  holds”

As an example, it holds:

$$\begin{aligned} \{b\}\emptyset\{a, b\}\{a, b, c\} &\in \text{pref}(P_{safe}) \\ \{a, b\}\{a, b\}\emptyset\{b, c\} &\in \text{pref}(P_{safe}) \\ \{b\}\{a, c\}\{a\}\{a, b, c\} &\in \text{BadPref}(P_{safe}) \\ \{b\}\{a, c\}\{a, c\}\{a\} &\in \text{BadPref}(P_{safe}) \end{aligned}$$

Questions:

- (a) Define an NFA  $\mathcal{A}$  such that  $\mathcal{L}(\mathcal{A}) = \text{MinBadPref}(P_{safe})$ .
- (b) Decide whether  $TS \models P_{safe}$  using the  $TS \otimes \mathcal{A}$  construction. Provide a counterexample if  $TS \not\models P_{safe}$ .

**Exercise 3****(2 + 2 points)**

Find nondeterministic Büchi automata that accept the following  $\omega$  regular languages:

(a)  $L_1 = \{\alpha \in \{A, B\}^\omega \mid \alpha \text{ contains } ABA \text{ infinitely often, but } AA \text{ only finitely often}\}$

(b)  $L_2 = \mathcal{L}((AB + C)^*((AA + B)C)^\omega + (A^*C)^\omega)$