

Introduction to Model Checking

Summer term 2010

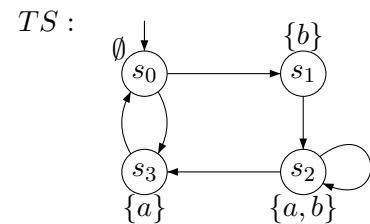
– Series 8 –

Hand in on June 23 before the exercise class.

Exercise 1

(0.5 + 1.5 + 2 + 1 + 1 + 2 points)

We consider the LTL formula $\varphi = \Box(a \rightarrow (\neg b U (a \wedge b)))$ over the set $AP = \{a, b\}$ of atomic propositions and want to check $TS \models \varphi$ wrt. the transition system outlined on the right.



- a) To check $TS \models \varphi$, convert $\neg\varphi$ into an equivalent LTL-formula ψ which is constructed according to the following grammar:

$$\varphi ::= \text{true} \mid \text{false} \mid a \mid b \mid \varphi \wedge \varphi \mid \neg\varphi \mid \bigcirc\varphi \mid \varphi U \varphi.$$

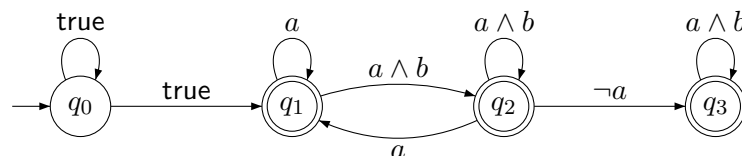
Then construct $\text{closure}(\psi)$.

- b) Give the elementary sets wrt. $\text{closure}(\psi)$!
- c) Construct the GNBA \mathcal{G}_ψ by providing its initial states, its acceptance set and its transition relation. Use the algorithm given in the lecture.
Hint: It suffices to provide the transition relation as a table.
- d) Now, construct an NBA $\mathcal{A}_{\neg\varphi}$ **directly** from $\neg\varphi$, i.e. without relying on \mathcal{G}_ψ .
Hint: Four states suffice!
- e) Construct $TS \otimes \mathcal{A}_{\neg\varphi}$.
- f) Use the Nested DFS algorithm from the lecture to check $TS \models \varphi$. Therefore sketch the algorithm's main steps and interpret its outcome!

Exercise 2

(2 points)

Consider the GNBA \mathcal{G} over the alphabet $\Sigma = 2^{\{a,b\}}$ and the set $\mathcal{F} = \{\{q_1, q_3\}, \{q_2\}\}$ of acceptance sets:



- a) Provide an LTL formula φ such that $\text{Word}(\varphi) = \mathcal{L}_\omega(\mathcal{G})$. Justify your answer!
- b) Depict an NBA \mathcal{A} with $\mathcal{L}_\omega(\mathcal{A}) = \mathcal{L}_\omega(\mathcal{G})$.

Exercise 3**(2 points)**

Let φ be an LTL-formula over a set of atomic propositions AP . Prove the following property:
For all elementary sets $B \subseteq \text{closure}(\varphi)$ and for all $B' \in \delta(B, B \cap AP)$, it holds:

$$\neg \bigcirc \psi \in B \iff \psi \notin B'.$$