

Introduction

Modelling parallel systems

Linear Time Properties

Regular Properties

Linear Temporal Logic (LTL)

Computation Tree Logic

 syntax and semantics of CTL

 expressiveness of CTL and LTL

 CTL model checking

 CTL with fairness



 counterexamples/witnesses, CTL^+ and CTL^*

Equivalences and Abstraction

LTL model checking problem:

PSPACE-complete and solvable in time

$$\mathcal{O}(\text{size}(\mathcal{T}) \cdot \exp(|\varphi|))$$

CTL model checking problem:

solvable in polynomial time

$$\mathcal{O}(\text{size}(\mathcal{T}) \cdot |\Phi|)$$

LTL model checking problem:

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LTL with fairness: $\mathcal{O}(\text{size}(\mathcal{T}) \cdot \exp(|\varphi|) \cdot |\text{fair}|)$

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CTL with fairness: $\mathcal{O}(\text{size}(\mathcal{T}) \cdot |\Phi| \cdot |\text{fair}|)$

Recall: LTL fairness assumptions

CTLFAIR4.4-2

are conjunctions of **LTL** formulas of the form

- unconditional fairness $\Box\Diamond\phi$
- strong fairness $\Box\Diamond\psi \rightarrow \Box\Diamond\phi$
- weak fairness $\Diamond\Box\psi \rightarrow \Box\Diamond\phi$

where ϕ, ψ are propositional formulas

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Reduction of $\models_{\text{fair}}$ to $\models$

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CTLFair4.4-2

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Reduction of $\models_{\text{fair}}$ to $\models$

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Reduction of $\models_{\text{fair}}$ to $\models$

$$\begin{aligned} \mathcal{T} \models_{\text{fair}} \varphi & \text{ iff } \pi \models \varphi \text{ for all fair paths } \pi \text{ in } \mathcal{T} \\ & \text{ iff for all paths } \pi \text{ in } \mathcal{T}: \\ & \quad \pi \models \text{fair} \rightarrow \varphi \end{aligned}$$

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CTLFAIR4.4-2

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Reduction of $\models_{\text{fair}}$ to $\models$, e.g., for $\text{fair} = \Box\Diamond a$

$\mathcal{T} \models_{\text{fair}} \varphi$ iff $\pi \models \varphi$ for all fair paths π in $\mathcal{T}$

iff for all paths π in $\mathcal{T}$:

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Recall: LTL fairness assumptions

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iff for all paths π in $\mathcal{T}$:

$$\pi \models \text{fair} \rightarrow \varphi \equiv \Diamond\Box\neg a \vee \varphi$$

conjunctions of “formulas” of the type

- unconditional fairness: $\Box\Diamond\Phi$
- strong fairness: $\Box\Diamond\Psi \rightarrow \Box\Diamond\Phi$
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where Ψ , Φ are CTL state formulas

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note: CTL fairness assumptions

- are not CTL (state or path) formulas
- just a syntactic formalism to specify fairness assumptions

conjunctions of “formulas” of the type

- unconditional fairness: $\Box\Diamond\Phi$
- strong fairness: $\Box\Diamond\Psi \rightarrow \Box\Diamond\Phi$
- weak fairness: $\Diamond\Box\Psi \rightarrow \Box\Diamond\Phi$

where Ψ , Φ are CTL state formulas

e.g., a strong CTL fairness assumption has the form:

$$\text{fair} = \bigwedge_{1 \leq j \leq k} (\Box\Diamond\Psi_j \rightarrow \Box\Diamond\Phi_j)$$

where Ψ_j , Φ_j are CTL state formulas

$$s \models_{\text{fair}} \text{true}$$

$$s \models_{\text{fair}} a \quad \text{iff} \quad a \in L(s)$$

$$s \models_{\text{fair}} \neg \Phi \quad \text{iff} \quad s \not\models_{\text{fair}} \Phi$$

$$s \models_{\text{fair}} \Phi_1 \wedge \Phi_2 \quad \text{iff} \quad s \models_{\text{fair}} \Phi_1 \text{ and } s \models_{\text{fair}} \Phi_2$$

$s \models_{\text{fair}} \text{true}$

$s \models_{\text{fair}} a$ iff $a \in L(s)$

$s \models_{\text{fair}} \neg\Phi$ iff $s \not\models_{\text{fair}} \Phi$

$s \models_{\text{fair}} \Phi_1 \wedge \Phi_2$ iff $s \models_{\text{fair}} \Phi_1$ and $s \models_{\text{fair}} \Phi_2$

$s \models_{\text{fair}} \exists\varphi$ iff there exists $\pi \in \text{Paths}(s)$ with
 $\pi \models_{\text{fair}}$ and $\pi \models_{\text{fair}} \varphi$

$$s \models_{\text{fair}} \text{true}$$

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$$s \models_{\text{fair}} \exists \varphi \quad \text{iff} \quad \text{there exists } \pi \in \text{Paths}(s) \text{ with} \\ \pi \models_{\text{fair}} \text{ and } \pi \models_{\text{fair}} \varphi$$

$$s \models_{\text{fair}} \forall \varphi \quad \text{iff} \quad \text{for all } \pi \in \text{Paths}(s): \\ \pi \models_{\text{fair}} \text{ implies } \pi \models_{\text{fair}} \varphi$$

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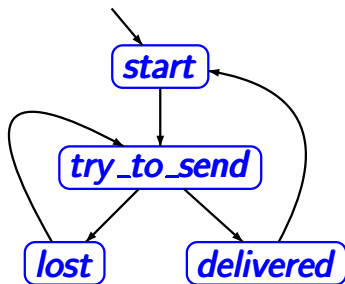
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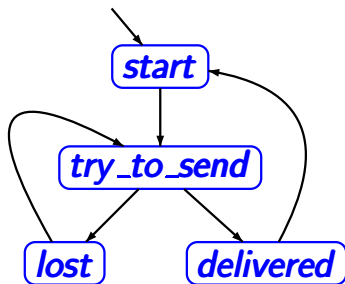
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$$\text{e.g., } s_0 s_1 s_2 \dots \models \Box \Diamond \Phi \quad \text{iff} \quad \exists i \geq 0 \text{ s.t. } s_i \models \Phi$$



CTL formula

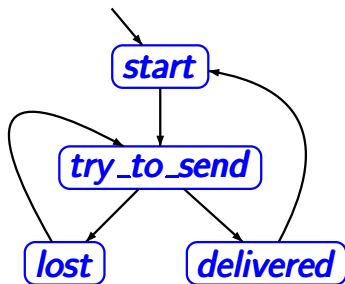
$$\Phi = \forall \square \forall \Diamond \textit{start}$$



CTL formula

$$\Phi = \forall \square \forall \Diamond \textit{start}$$

$$\mathcal{T} \not\models \Phi$$



CTL formula

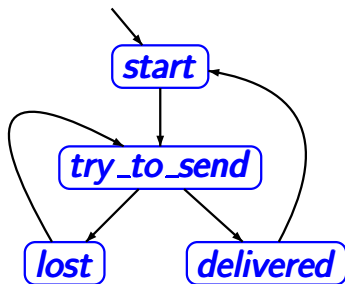
$$\Phi = \forall \square \forall \Diamond \textit{start}$$

$$\mathcal{T} \not\models \Phi$$

$$\mathcal{T} \models_{\textit{ufair}} \Phi$$

unconditional CTL fairness assumption:

$$\textit{ufair} = \square \Diamond \textit{delivered}$$



CTL formula

$$\Phi = \forall \square \forall \Diamond \text{start}$$

$$\mathcal{T} \not\models \Phi$$

$$\mathcal{T} \models_{\text{ufair}} \Phi$$

$$\mathcal{T} \models_{\text{sfair}} \Phi$$

unconditional CTL fairness assumption:

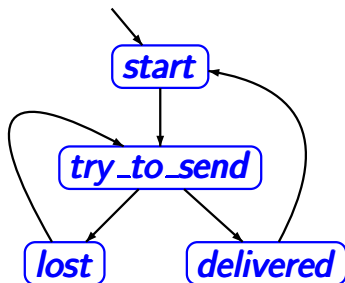
$$\text{ufair} = \square \Diamond \text{delivered}$$

strong CTL fairness assumption:

$$\text{sfair} = \square \Diamond \text{try_to_send} \rightarrow \square \Diamond \text{delivered}$$

Simple communication protocol

CTLFAIR4.4-6



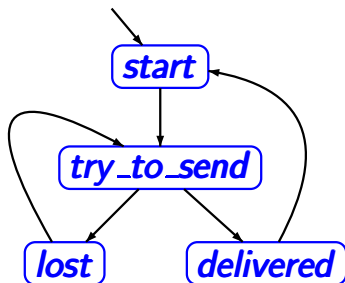
$$\phi = \forall \square \forall \diamond \textit{start}$$

$$\mathcal{T} \models_{\textit{ufair}} \phi \quad ?$$

unconditional fairness: $\textit{ufair} = \square \diamond \exists \bigcirc \textit{start}$

Simple communication protocol

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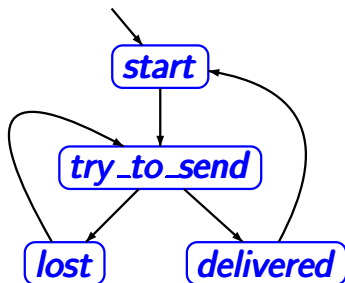
unconditional fairness: $\textit{ufair} = \square \diamond \boxed{\exists \bigcirc \textit{start}}$



$$\textit{Sat}(\exists \bigcirc \textit{start}) = \{\textit{delivered}\}$$

Simple communication protocol

CTLFAIR4.4-6



$$\phi = \forall \square \forall \diamond \text{start}$$

$$\mathcal{T} \models_{\text{fair}} \phi \quad ?$$

unconditional fairness: $\text{fair} = \square \diamond \boxed{\exists \bigcirc \text{start}}$

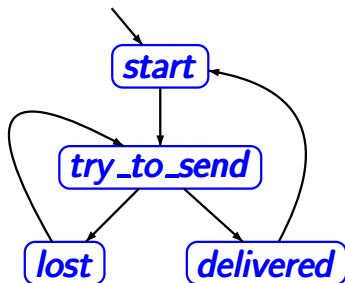


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$$\text{fair} \triangleq \square \diamond \text{delivered}$$

Simple communication protocol

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$$\phi = \forall \square \forall \diamond \text{start}$$

$$\mathcal{T} \models_{\text{ufair}} \phi \quad \checkmark$$

unconditional fairness: $\text{ufair} = \square \diamond \exists \bigcirc \text{start}$

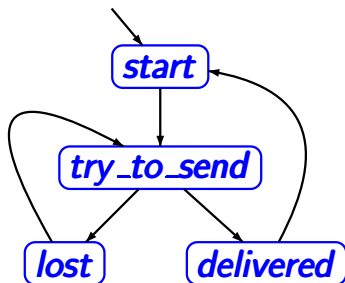


$$\text{Sat}(\exists \bigcirc \text{start}) = \{\text{delivered}\}$$

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$$\Phi = \forall \Box \forall \Diamond \text{start}$$

$$\mathcal{T} \models_{\text{ufair}} \Phi \quad \checkmark$$

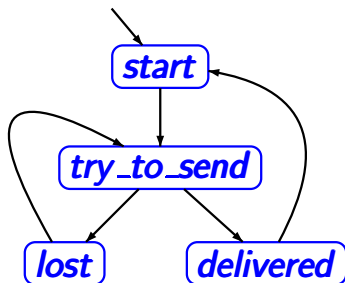
$$\mathcal{T} \models_{\text{wfair}} \Phi \quad ?$$

unconditional fairness: $\text{ufair} = \Box \Diamond \exists \bigcirc \text{start}$

weak fairness: $\text{wfair} = \Diamond \Box \exists \bigcirc \text{delivered} \rightarrow \Box \Diamond \text{delivered}$

Simple communication protocol

CTLFAIR4.4-6



$$\Phi = \forall \Box \forall \Diamond \text{start}$$

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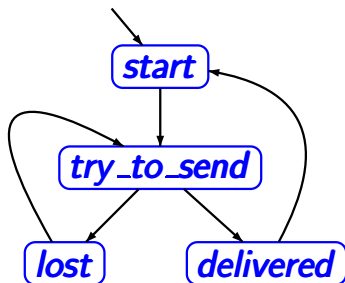
unconditional fairness: $\text{ufair} = \Box \Diamond \exists \bigcirc \text{start}$

weak fairness: $\text{wfair} = \Diamond \Box [\exists \bigcirc \text{delivered}] \rightarrow \Box \Diamond \text{delivered}$

$$\text{Sat}(\exists \bigcirc \text{delivered}) = \{\text{try_to_send}\}$$

Simple communication protocol

CTLFAIR4.4-6



$$\Phi = \forall \Box \forall \Diamond \text{start}$$

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unconditional fairness: $\text{ufair} = \Box \Diamond \exists \bigcirc \text{start}$

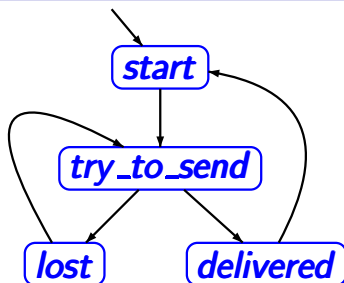
weak fairness: $\text{wfair} = \Diamond \Box \boxed{\exists \bigcirc \text{delivered}} \rightarrow \Box \Diamond \text{delivered}$

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Simple communication protocol

CTLFAIR4.4-6



$$\Phi = \forall \Box \forall \Diamond \text{start}$$

$$\mathcal{T} \models_{\text{ufair}} \Phi \quad \checkmark$$

$$\mathcal{T} \models_{\text{wfair}} \Phi \quad \text{wrong}$$

unconditional fairness: $\text{ufair} = \Box \Diamond \exists \bigcirc \text{start}$

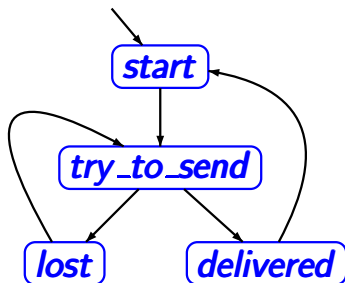
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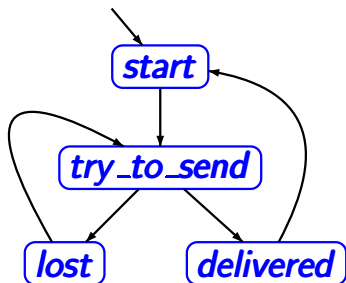
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strong fairness: $\text{sfair} = \square \Diamond \exists \bigcirc \text{delivered} \rightarrow \square \Diamond \text{delivered}$

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CTLFAIR4.4-6



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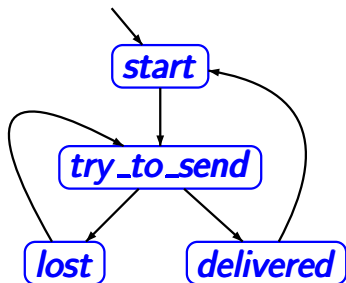
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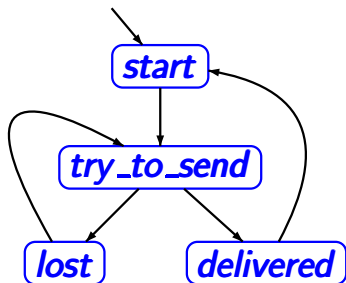
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Simple communication protocol

CTLFair4.4-6



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$$\text{Sat}(\exists \bigcirc \text{delivered}) = \{\text{try_to_send}\}$$

$$\text{sfair} \triangleq \Box \Diamond \text{try_to_send} \rightarrow \Box \Diamond \text{delivered}$$

Correct or wrong?

CTLFAIR4.4-7

If $s \models \forall \Diamond a$ where $a \in AP$ then $s \models_{fair} \forall \Diamond a$

Correct or wrong?

CTLFAIR4.4-7

If $s \models \forall \Diamond a$ where $a \in AP$ then $s \models_{fair} \forall \Diamond a$

correct.

Correct or wrong?

CTLFAIR4.4-7

If $s \models \forall \Diamond a$ where $a \in AP$ then $s \models_{\text{fair}} \forall \Diamond a$

correct. Note that:

$s \models \forall \varphi \implies \text{for all } \pi \in \text{Paths}(s): \pi \models \varphi$

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$\implies s \models_{fair} \forall \varphi$

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CTLFAIR4.4-7

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correct.

If $s \models \exists \Diamond a$ where $a \in AP$ then $s \models_{\text{fair}} \exists \Diamond a$

Correct or wrong?

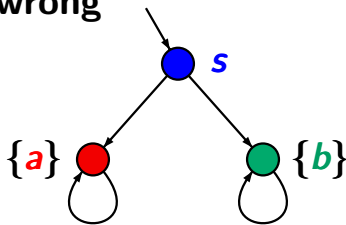
CTLFAIR4.4-7

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correct.

If $s \models \exists \Diamond a$ where $a \in AP$ then $s \models_{\text{fair}} \exists \Diamond a$

wrong



$\text{fair} = \Box \Diamond b$

Correct or wrong?

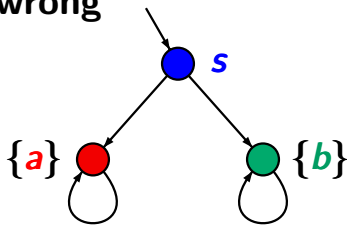
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$fair = \Box \Diamond b$

just one fair path ● ● ● ● . . .

Correct or wrong?

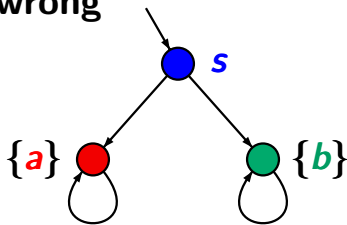
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$fair = \Box \Diamond b$

$s \not\models_{fair} \exists \Diamond a$

just one fair path ● ● ● ● . . .

Correct or wrong?

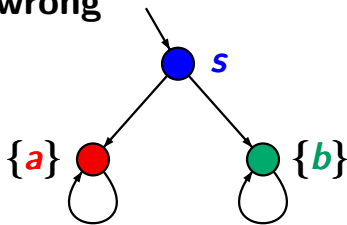
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wrong



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$s \models \exists \Diamond a$

just one fair path ● ● ● ● . . .

Correct or wrong?

CTLFAIR4.4-7A

If $s \models \forall \Diamond a$ where $a \in AP$ then $s \models_{\text{fair}} \forall \Diamond a$

correct.

Does the same condition hold if a is replaced with an arbitrary state formula ?

Correct or wrong?

CTLFAIR4.4-8

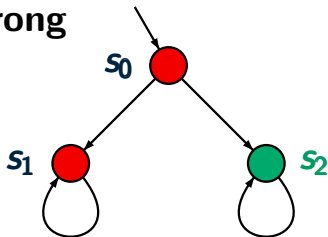
If $s \models \forall \Diamond \exists \Box a$ then $s \models_{\text{fair}} \forall \Diamond \exists \Box a$

Correct or wrong?

CTLFAIR4.4-8

If $s \models \forall \Diamond \exists \Box a$ then $s \models_{\text{fair}} \forall \Diamond \exists \Box a$

wrong



● = { b }

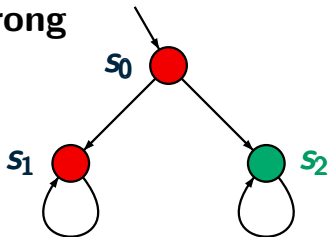
● = { a }

Correct or wrong?

CTLFAIR4.4-8

If $s \models \forall \Diamond \exists \Box a$ then $s \models_{\text{fair}} \forall \Diamond \exists \Box a$

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● = { b }

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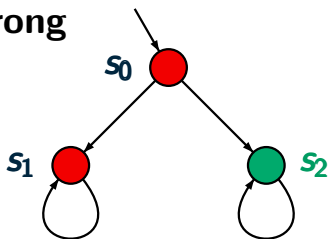
$$\text{Sat}(\exists \Box a) = \{s_0, s_1\}$$

Correct or wrong?

CTLFAIR4.4-8

If $s \models \forall \Diamond \exists \Box a$ then $s \models_{\text{fair}} \forall \Diamond \exists \Box a$

wrong



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● = { a }

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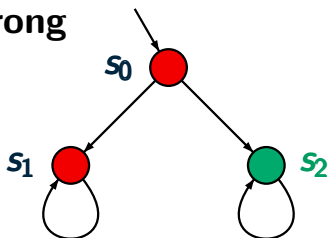
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Correct or wrong?

CTLFAIR4.4-8

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wrong



● = { b }

● = { a }

$\text{fair} = \Box \Diamond b$

$\text{Sat}(\exists \Box a) = \{s_0, s_1\}$

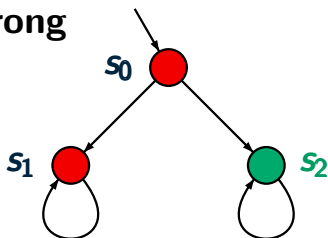
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Correct or wrong?

CTLFAIR4.4-8

If $s \models \forall \Diamond \exists \Box a$ then $s \models_{\text{fair}} \forall \Diamond \exists \Box a$

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$\text{Sat}(\exists \Box a) = \{s_0, s_1\}$

$\text{Sat}_{\text{fair}}(\exists \Box a) = \emptyset$

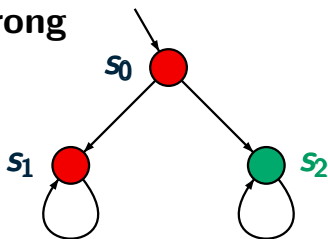
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Correct or wrong?

CTLFAIR4.4-8

If $s \models \forall \Diamond \exists \Box a$ then $s \models_{\text{fair}} \forall \Diamond \exists \Box a$

wrong



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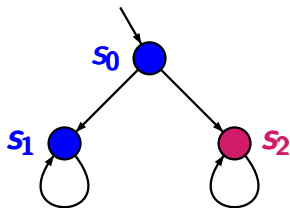
$\text{Sat}_{\text{fair}}(\forall \Diamond \exists \Box a) = \emptyset$

Sat_{fair}($\exists \Box true$) = ?

CTLFair4.4-11

$Sat_{fair}(\exists \Box true) = ?$

CTLFAIR4.4-11



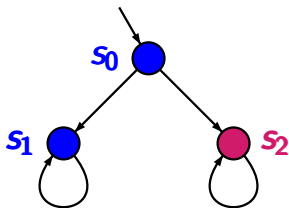
● = {*a*}

● = ∅

fair = $\Box \Diamond a$

$$Sat_{fair}(\exists \Box true) = ?$$

CTLFAIR4.4-11



$$\text{pink circle} = \{a\}$$

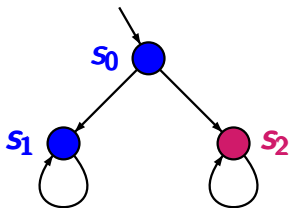
$$\text{blue circle} = \emptyset$$

$$fair = \Box \Diamond a$$

$$Sat_{fair}(\exists \Box true) = ?$$

$$Sat_{fair}(\exists \Box true) = ?$$

CTLFAIR4.4-11



$$\text{pink circle} = \{a\}$$

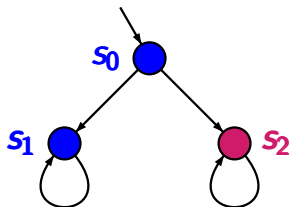
$$\text{blue circle} = \emptyset$$

$$fair = \Box \Diamond a$$

$$Sat_{fair}(\exists \Box true) = \{s_0, s_2\}$$

$$Sat_{fair}(\exists \Box true) = ?$$

CTLFAIR4.4-11



$$\text{pink circle} = \{a\}$$

$$\text{blue circle} = \emptyset$$

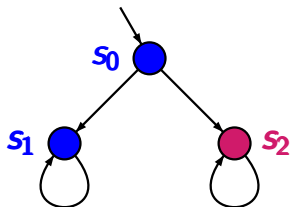
$$fair = \Box \Diamond a$$

$$Sat_{fair}(\exists \Box true) = \{s_0, s_2\}$$

$Sat_{fair}(\exists \Box true)$ = set of states s that have at least one fair path

$$Sat_{fair}(\exists \Box true) = ?$$

CTLFAIR4.4-11



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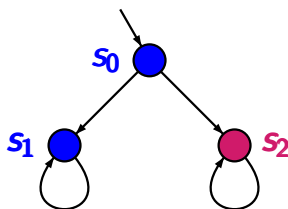
$$Sat_{fair}(\exists \Box true) = \{s_0, s_2\}$$

$Sat_{fair}(\exists \Box true)$ = set of states s that have at least one fair path

$$= \{s : \exists \pi \in Paths(s) \text{ s.t. } \pi \models fair\}$$

$$Sat_{fair}(\exists \Box true) = ?$$

CTLFAIR4.4-11



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$Sat_{fair}(\exists \Box true)$ = set of states s that have at least one fair path

$$= \{s : \exists \pi \in Paths(s) \text{ s.t. } \pi \models fair\}$$

$fair$ is realizable iff

$$Sat_{fair}(\exists \Box true) \supseteq \text{set of all reachable states}$$

Model checking problem for FairCTL

CTLFAIR4.4-12

Model checking problem for FairCTL

CTLFAIR4.4-12

given: finite transition system $\mathcal{T}$
 CTL formula Φ
 CTL fairness assumption *fair*

question: does $\mathcal{T} \models_{\text{fair}} \Phi$ hold ?

Model checking problem for FairCTL

CTLFAIR4.4-12

given: finite transition system $\mathcal{T}$
 CTL formula Φ
 CTL fairness assumption *fair*, e.g.,

$$\textit{fair} = \bigwedge_{1 \leq i \leq k} \Box \Diamond \psi_{i,1} \rightarrow \Box \Diamond \psi_{i,2}$$

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question: does $\mathcal{T} \models_{\textit{fair}} \Phi$ hold ?

for simplicity:

we suppose that Φ is in **existential normal form**,
i.e., a $\forall$ -free CTL formula with temporal modalities

$$\exists \bigcirc, \exists \bigcup, \exists \Box$$

given: finite transition system $\mathcal{T}$
 CTL formula Φ in $\exists$ -normal form
 CTL fairness assumption *fair*, e.g.,

$$\textit{fair} = \bigwedge_{1 \leq i \leq k} \Box \Diamond \psi_{i,1} \rightarrow \Box \Diamond \psi_{i,2}$$

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preprocessing: apply a standard CTL model checker
 to evaluate the CTL state subformulas of *fair*

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preprocessing: apply a standard CTL model checker
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- compute $\text{Sat}(\Psi_{i,1})$ and $\text{Sat}(\Psi_{i,2})$
- replace $\Psi_{i,1}$ and $\Psi_{i,2}$ with fresh atomic propositions b_i and c_i , respectively

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CTL formula Φ in $\exists$ -normal form
CTL fairness assumption *fair*, e.g.,

$$\textit{fair} = \bigwedge_{1 \leq i \leq k} \Box \Diamond b_i \rightarrow \Box \Diamond c_i \text{ with } b_i, c_i \in AP$$

question: does $\mathcal{T} \models_{\textit{fair}} \Phi$ hold ?

preprocessing: apply a standard CTL model checker to evaluate the CTL state subformulas of *fair*

- compute $\textit{Sat}(\Psi_{i,1})$ and $\textit{Sat}(\Psi_{i,2})$
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1. ... preprocessing ...

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 CTL formula Φ in $\exists$ -normal form
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question: does $\mathcal{T} \models_{\text{fair}} \Phi$ hold ?

1. ... preprocessing ...
2. Build the parse tree of Φ and process it in bottom-up-manner.

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1. ... preprocessing ...
2. Build the parse tree of Φ and process it in bottom-up-manner. Treatment of:
 - *true*, $a \in AP$, $\wedge$, $\neg$: as for **standard CTL**

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 - $\exists O$, $\exists U$: via **standard CTL** model checking

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 - *true*, $a \in AP$, $\wedge$, $\neg$: as for **standard CTL**
 - $\exists O$, $\exists U$: via **standard CTL** model checking
 - $\exists \square$: via analysis of **SCCs**

recursive computation of the fair satisfaction sets:

$$Sat_{fair}(\Psi) = \{s \in S : s \models_{fair} \Psi\}$$

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simple cases: $\Psi = true$ or $\Psi = a \in AP$ or the outer most operator of Ψ is a negation or conjunction:

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$$Sat_{fair}(\Psi) = \{s \in S : s \models_{fair} \Psi\}$$

simple cases: $\Psi = true$ or $\Psi = a \in AP$ or the outer most operator of Ψ is a negation or conjunction:

$$Sat_{fair}(true) = S$$

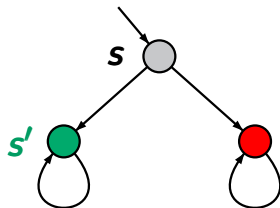
$$Sat_{fair}(a) = \{s \in S : a \in L(s)\}$$

$$Sat_{fair}(\neg \Psi) = S \setminus Sat_{fair}(\Psi)$$

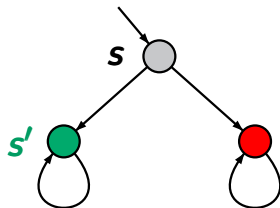
$$Sat_{fair}(\Psi_1 \wedge \Psi_2) = Sat_{fair}(\Psi_1) \cap Sat_{fair}(\Psi_2)$$

given: finite transition system $\mathcal{T}$
 CTL formula Φ in $\exists$ -normal form
 CTL fairness assumption *fair*
question: does $\mathcal{T} \models_{\text{fair}} \Phi$ hold ?

1. ... preprocessing ...
2. Build the parse tree of Φ and process it in bottom-up-manner. Treatment of:
 - *true*, $a \in AP$, $\wedge$, $\neg$: as for standard CTL
 - $\exists O$, $\exists U$: via **standard CTL model checking**
 - $\exists \square$: via analysis of SCCs

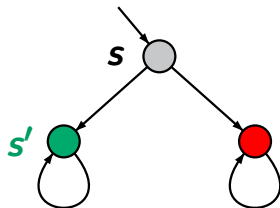


$$\textit{fair} = \Box\Diamond \textit{red}$$



$$\textit{fair} = \Box\Diamond \textit{red}$$

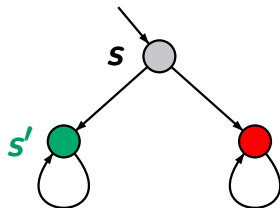
$$s \not\models_{\textit{fair}} \exists\bigcirc \textit{green}$$



$$fair = \Box\Diamond red$$

$$s \not\models_{fair} \exists\bigcirc green$$

$$\text{as } s' \not\models_{fair} \exists\Box true$$



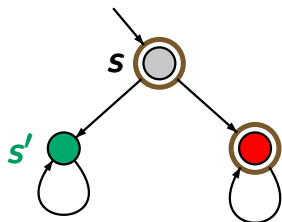
$$fair = \Box\Diamond red$$

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$$\text{as } s' \not\models_{fair} \exists\Box true$$

introduce an additional atomic proposition a_{fair}
 s.t. for all states s :

$$a_{fair} \in L(s) \text{ iff } s \models_{fair} \exists\Box true$$



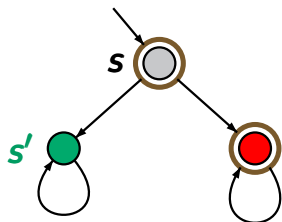
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This yields that for all $b \in AP$ and all states s :

$$s \models_{fair} \exists\bigcirc b \text{ iff } s \models \exists\bigcirc(b \wedge a_{fair})$$

introduce an additional atomic proposition a_{fair} s.t.

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This yields that for all $b, c \in AP$ and all states s :

$$s \models_{fair} \exists\bigcirc b \quad \text{iff} \quad s \models \exists\bigcirc(b \wedge a_{fair})$$

$$s \models_{fair} \exists(c \mathbf{U} b) \quad \text{iff} \quad ?$$

introduce an additional atomic proposition a_{fair} s.t.

$$a_{fair} \in L(s) \text{ iff } s \models_{fair} \exists\Box true$$

This yields that for all $b, c \in AP$ and all states s :

$$\begin{array}{ll} s \models_{fair} \exists\bigcirc b & \text{iff } s \models \exists\bigcirc(b \wedge a_{fair}) \\ s \models_{fair} \exists(c \mathbf{U} b) & \text{iff } s \models \exists(c \mathbf{U}(b \wedge a_{fair})) \end{array}$$

introduce an additional atomic proposition a_{fair} s.t.

$$a_{fair} \in L(s) \text{ iff } s \models_{fair} \exists\Box true$$

This yields that for all $b, c \in AP$ and all states s :

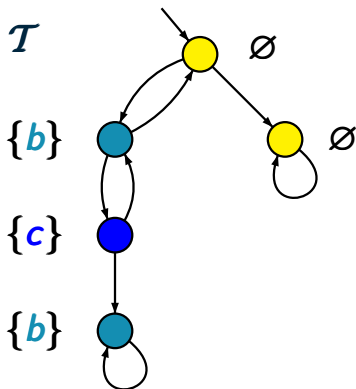
$$\begin{aligned} s \models_{fair} \exists\bigcirc b & \quad \text{iff} \quad s \models \exists\bigcirc(b \wedge a_{fair}) \\ s \models_{fair} \exists(c \mathbf{U} b) & \quad \text{iff} \quad s \models \exists(c \mathbf{U}(b \wedge a_{fair})) \end{aligned}$$

hence: treatment of $\exists\bigcirc$ and $\exists\mathbf{U}$ for FairCTL via

- special methods to compute $Sat_{fair}(\exists\Box true)$
- standard CTL model checking for $\exists\bigcirc$ and $\exists\mathbf{U}$

Example: treatment of $\exists\Diamond$

CTLFAIR4.4-15

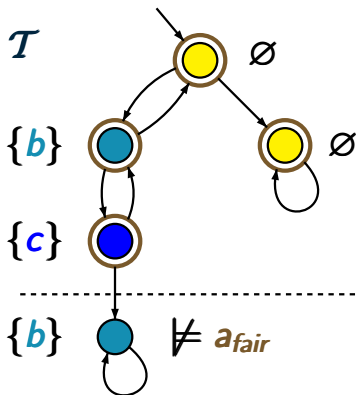


CTL formula $\exists\Diamond c$

strong fairness assumption: $\text{fair} = \Box\Diamond b \rightarrow \Box\Diamond c$

Example: treatment of $\exists\Diamond$

CTLFAIR4.4-15



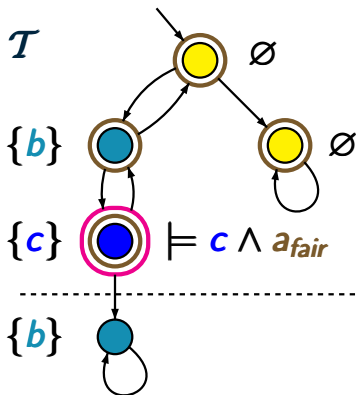
CTL formula $\exists\Diamond c$

$\downarrow$
 $\exists\Diamond (c \wedge a_{fair})$

strong fairness assumption: $fair = \Box\Diamond b \rightarrow \Box\Diamond c$

Example: treatment of $\exists\Diamond$

CTLFAIR4.4-15

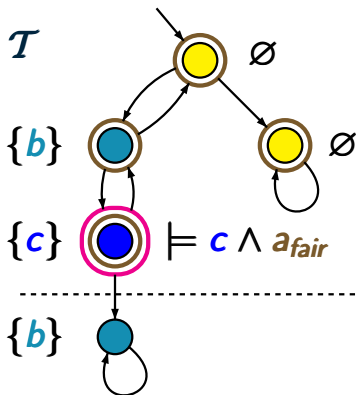


CTL formula $\exists\Diamond c$

$\downarrow$

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strong fairness assumption: $fair = \Box\Diamond b \rightarrow \Box\Diamond c$



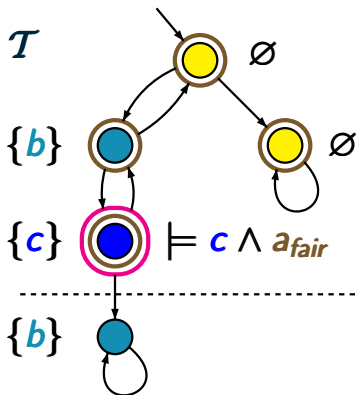
CTL formula $\exists\Diamond c$

$\downarrow$

$\exists\Diamond (c \wedge a_{fair})$

strong fairness assumption: $fair = \Box\Diamond b \rightarrow \Box\Diamond c$

$\mathcal{T} \models \exists\Diamond (c \wedge a_{fair})$



CTL formula $\exists\Diamond c$

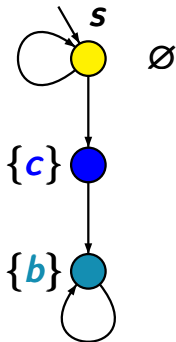
$$\downarrow$$

$$\exists\Diamond (c \wedge a_{fair})$$

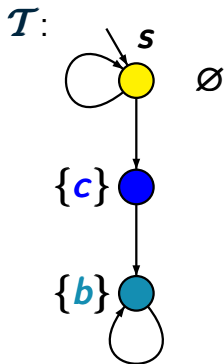
strong fairness assumption: $fair = \Box\Diamond b \rightarrow \Box\Diamond c$

$$\mathcal{T} \models \exists\Diamond (c \wedge a_{fair}) \implies \mathcal{T} \models_{fair} \exists\Diamond c$$

$\mathcal{T}$:

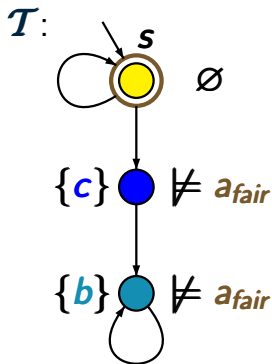


$$\mathcal{T} \models \exists(\neg b \cup c)$$



strong fairness assumption: $\text{fair} = \Box \Diamond b \rightarrow \Box \Diamond c$

$$\mathcal{T} \models \exists(\neg b \cup c)$$



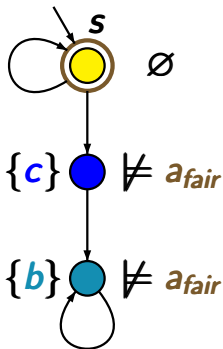
strong fairness assumption: $fair = \Box \Diamond b \rightarrow \Box \Diamond c$

$$\mathcal{T} \models \exists(\neg b \cup c)$$

Example: treatment of $\exists U$

CTLFAIR4.4-17

$\mathcal{T}$:



$$\text{Sat}(c \wedge a_{fair}) = \emptyset$$

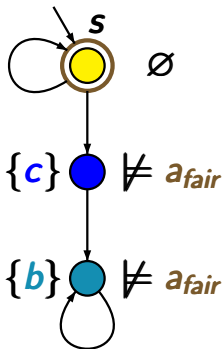
strong fairness assumption: $fair = \Box \Diamond b \rightarrow \Box \Diamond c$

$$\mathcal{T} \models \exists(\neg b U c)$$

Example: treatment of $\exists U$

CTLFAIR4.4-17

$\mathcal{T}$:



$$s \not\models \exists(\neg b \text{ U } (c \wedge a_{fair}))$$

$\Uparrow$

$$Sat(c \wedge a_{fair}) = \emptyset$$

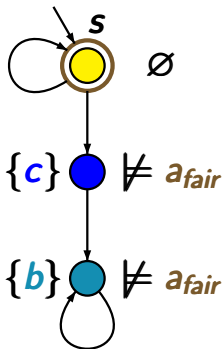
strong fairness assumption: $fair = \Box \Diamond b \rightarrow \Box \Diamond c$

$$\mathcal{T} \models \exists(\neg b \text{ U } c)$$

Example: treatment of $\exists U$

CTLFair4.4-17

$\mathcal{T}$:



$$s \not\models_{fair} \exists(\neg b U c)$$

$\Uparrow$

$$s \not\models \exists(\neg b U (c \wedge a_{fair}))$$

$\Uparrow$

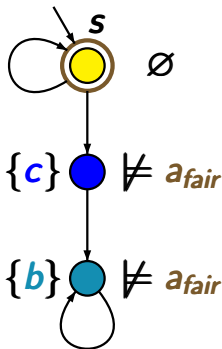
$$Sat(c \wedge a_{fair}) = \emptyset$$

strong fairness assumption: $fair = \Box \Diamond b \rightarrow \Box \Diamond c$

$$\mathcal{T} \models \exists(\neg b U c)$$

Example: treatment of $\exists U$

CTLFAIR4.4-17

 $\mathcal{T}$:

$$s \not\models_{\text{fair}} \exists(\neg b U c)$$

 $\Uparrow$

$$s \not\models \exists(\neg b U (c \wedge a_{\text{fair}}))$$

 $\Uparrow$

$$\text{Sat}(c \wedge a_{\text{fair}}) = \emptyset$$

strong fairness assumption: $\text{fair} = \Box \Diamond b \rightarrow \Box \Diamond c$

$$\mathcal{T} \models \exists(\neg b U c), \quad \text{but } \mathcal{T} \not\models_{\text{fair}} \exists(\neg b U c)$$

Correct or wrong?

CTLFAIR4.4-16

$$s \models_{\text{fair}} \exists \bigcirc \exists (c \cup b) \quad \text{iff} \quad s \models \exists \bigcirc \exists (c \cup (b \wedge a_{\text{fair}}))$$

Correct or wrong?

CTLFAIR4.4-16

$$s \models_{\text{fair}} \exists \bigcirc \exists (c \cup b) \quad \text{iff} \quad s \models \exists \bigcirc \exists (c \cup (b \wedge a_{\text{fair}}))$$

correct.

$$s \models_{\text{fair}} \exists \bigcirc \exists (c \cup b) \quad \text{iff} \quad s \models \exists \bigcirc \exists (c \cup (b \wedge a_{\text{fair}}))$$

correct. Note that:

if $s_0 s_1 \dots s_{n-1} s_n$ is a path fragment from $s_0 = s$ s.t.
 $s_n \models a_{\text{fair}}$ then $s_0, s_1, \dots, s_{n-1} \models a_{\text{fair}}$.

$$s \models_{\text{fair}} \exists \bigcirc \exists (c \text{ U } b) \quad \text{iff} \quad s \models \exists \bigcirc \exists (c \text{ U } (b \wedge a_{\text{fair}}))$$

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$$s \models \exists \bigcirc \exists (c \text{ U } (b \wedge a_{\text{fair}}))$$

$$\iff s \models \exists \bigcirc \exists ((c \wedge a_{\text{fair}}) \text{ U } (b \wedge a_{\text{fair}}))$$

$$s \models_{\text{fair}} \exists \bigcirc \exists (c \text{ U } b) \quad \text{iff} \quad s \models \exists \bigcirc \exists (c \text{ U } (b \wedge a_{\text{fair}}))$$

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if $s_0 s_1 \dots s_{n-1} s_n$ is a path fragment from $s_0 = s$ s.t.
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$$\iff s \models_{\text{fair}} \exists \bigcirc \exists (c \text{ U } b)$$

Correct or wrong?

CTLFAIR4.4-16

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Correct or wrong?

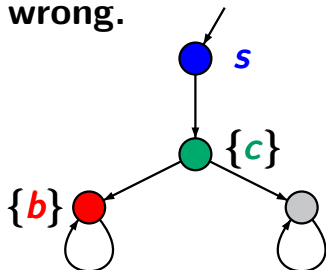
CTLFAIR4.4-16

$$s \models_{\text{fair}} \exists \bigcirc \exists (c \cup b) \quad \text{iff} \quad s \models \exists \bigcirc \exists (c \cup (b \wedge a_{\text{fair}}))$$

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$$s \models_{\text{fair}} \exists \bigcirc \exists (c \cup b) \quad \text{iff} \quad s \models \exists \bigcirc (\exists (c \cup b) \wedge a_{\text{fair}})$$

wrong.



$$\text{fair} = \Box \Diamond \text{gray}$$

Correct or wrong?

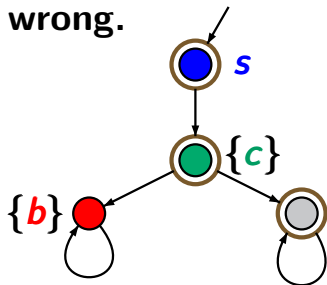
CTLFAIR4.4-16

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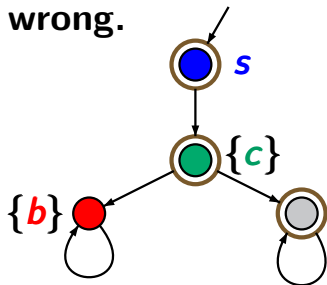
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wrong.



$$\text{fair} = \Box \Diamond \text{gray}$$

$$\text{Sat}_{\text{fair}}(\exists (c \cup b)) = \emptyset$$

Correct or wrong?

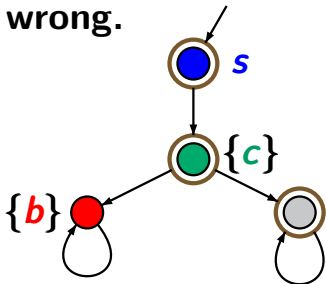
CTLFAIR4.4-16

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$$s \not\models_{\text{fair}} \exists \bigcirc \exists (c \cup b)$$

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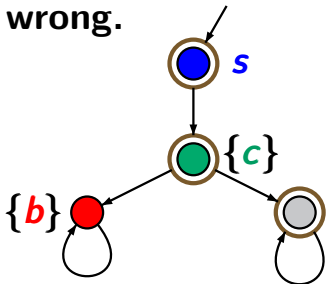
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$$s \models \exists \bigcirc (\exists (c \cup b) \wedge a_{\text{fair}})$$

Correct or wrong?

CTLFAIR4.4-23

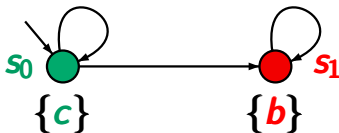
$$s \models_{\text{fair}} \exists \Box c \quad \text{iff} \quad s \models \exists \Box (c \wedge a_{\text{fair}})$$

Correct or wrong?

CTLFAIR4.4-23

$$s \models_{\text{fair}} \exists \Box c \text{ iff } s \models \exists \Box (c \wedge a_{\text{fair}})$$

wrong.



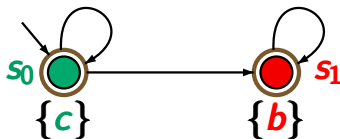
$$\text{fair} = \Box \Diamond b$$

Correct or wrong?

CTLFAIR4.4-23

$$s \models_{\text{fair}} \exists \Box c \text{ iff } s \models \exists \Box (c \wedge a_{\text{fair}})$$

wrong.



$$\text{fair} = \Box \Diamond b$$

$$s_0 \models a_{\text{fair}}$$

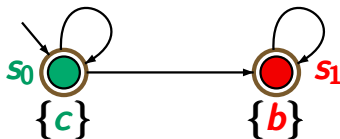
$$s_1 \models a_{\text{fair}}$$

Correct or wrong?

CTLFAIR4.4-23

$$s \models_{\text{fair}} \exists \Box c \text{ iff } s \models \exists \Box (c \wedge a_{\text{fair}})$$

wrong.



$$\text{fair} = \Box \Diamond b$$

$$s_0 \models a_{\text{fair}}$$

$$s_1 \models a_{\text{fair}}$$

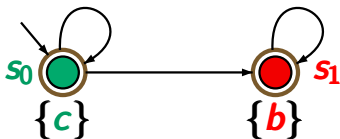
regard state $s = s_0$:

Correct or wrong?

CTLFAIR4.4-23

$$s \models_{\text{fair}} \exists \Box c \text{ iff } s \models \exists \Box (c \wedge a_{\text{fair}})$$

wrong.



$$\text{fair} = \Box \Diamond b$$

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regard state $s = s_0$:

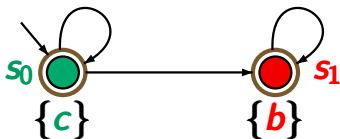
$$s \models \exists \Box (c \wedge a_{\text{fair}}),$$

Correct or wrong?

CTLFAIR4.4-23

$$s \models_{\text{fair}} \exists \Box c \quad \text{iff} \quad s \models \exists \Box (c \wedge a_{\text{fair}})$$

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$$s_1 \models a_{\text{fair}}$$

regard state $s = s_0$:

$$s \models \exists \Box (c \wedge a_{\text{fair}}),$$

↑

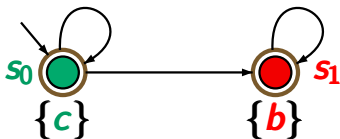
$$\text{path } \pi = s_0 s_0 s_0 s_0 \dots \models \Box (c \wedge a_{\text{fair}})$$

Correct or wrong?

CTLFAIR4.4-23

$$s \models_{\text{fair}} \exists \Box c \quad \text{iff} \quad s \models \exists \Box (c \wedge a_{\text{fair}})$$

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$$\text{fair} = \Box \Diamond b$$

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regard state $s = s_0$:

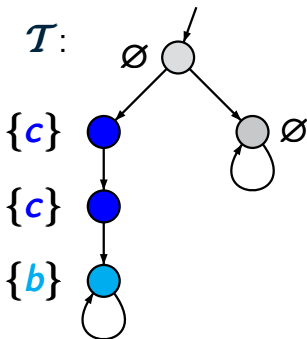
$$s \models \exists \Box (c \wedge a_{\text{fair}}), \quad \text{but} \quad s \not\models_{\text{fair}} \exists \Box c$$

$$\begin{array}{c} \uparrow \\ \text{path } \pi = s_0 s_0 s_0 s_0 \dots \models \Box (c \wedge a_{\text{fair}}) \end{array}$$

given: finite transition system $\mathcal{T}$
 CTL formula Φ in $\exists$ -normal form
 CTL fairness assumption *fair*
question: does $\mathcal{T} \models_{\text{fair}} \Phi$ hold ?

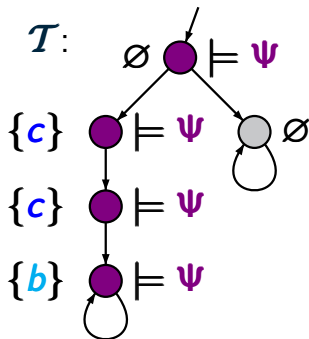
1. ... preprocessing ...
2. Build the parse tree of Φ and process it in bottom-up-manner. Treatment of:
 - *true*, $a \in AP$, $\wedge$, $\neg$: as for standard CTL
 - $\exists O$, $\exists U$: via standard CTL model checking
 - $\exists \square$: via analysis of **SCCs**

fair = $\Box \Diamond b \rightarrow \Box \Diamond c$, CTL state formula Ψ



$\mathcal{T} \models_{\text{fair}} \exists \Box \Psi$?

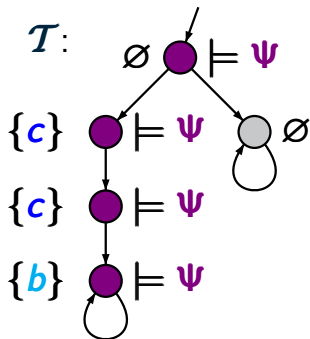
fair = $\Box \Diamond b \rightarrow \Box \Diamond c$, CTL state formula Ψ



$\mathcal{T} \models_{\text{fair}} \exists \Box \Psi$?

1. calculate $\text{Sat}_{\text{fair}}(\Psi)$

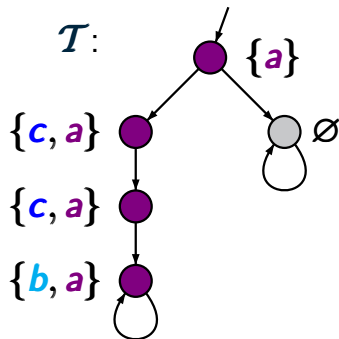
fair = $\Box \Diamond b \rightarrow \Box \Diamond c$, CTL state formula Ψ



$\mathcal{T} \models_{\text{fair}} \exists \Box \Psi$?

1. calculate $\text{Sat}_{\text{fair}}(\Psi)$
2. replace Ψ with a fresh atomic proposition $a = a_{\Psi}$

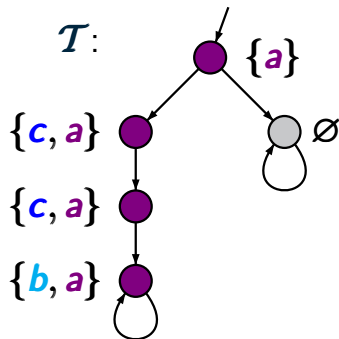
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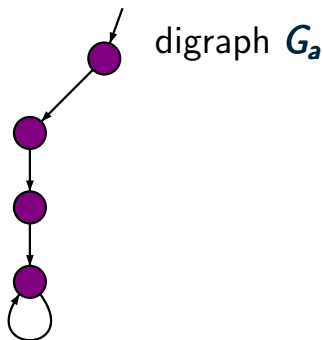
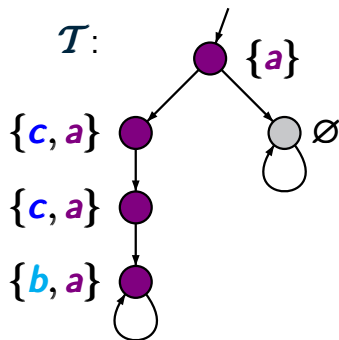
fair = $\Box \Diamond b \rightarrow \Box \Diamond c$, CTL state formula Ψ



$\mathcal{T} \models_{\text{fair}} \exists \Box \Psi$?

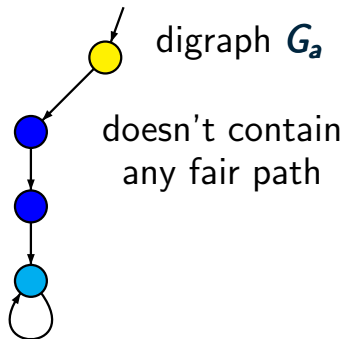
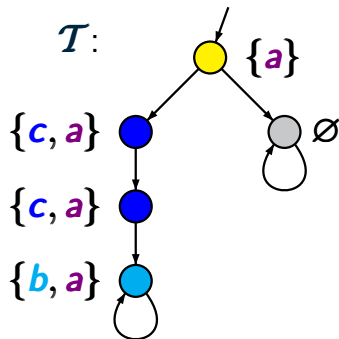
1. calculate $\text{Sat}_{\text{fair}}(\Psi)$
2. replace Ψ with a fresh atomic proposition $a = a_{\Psi}$
3. calculate $\text{Sat}_{\text{fair}}(\exists \Box a)$

fair = $\Box\Diamond b \rightarrow \Box\Diamond c$, CTL state formula Ψ



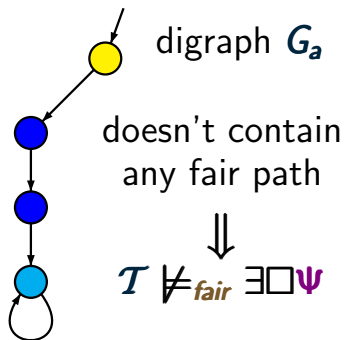
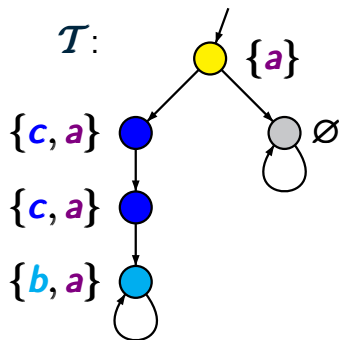
1. calculate $Sat_{fair}(\Psi)$
2. replace Ψ with a fresh atomic proposition $a = a_\Psi$
3. calculate $Sat_{fair}(\exists\Box a)$

fair = $\Box\Diamond b \rightarrow \Box\Diamond c$, CTL state formula Ψ



1. calculate $Sat_{fair}(\Psi)$
2. replace Ψ with a fresh atomic proposition $a = a_\Psi$
3. calculate $Sat_{fair}(\exists\Box a)$

fair = $\Box \Diamond b \rightarrow \Box \Diamond c$, CTL state formula Ψ



1. calculate $\text{Sat}_{\text{fair}}(\Psi)$
2. replace Ψ with a fresh atomic proposition $a = a_\Psi$
3. calculate $\text{Sat}_{\text{fair}}(\exists \Box a) = \emptyset$

given: finite TS $\mathcal{T}$, atomic proposition a
CTL fairness assumption *fair*

goal: compute $Sat_{fair}(\exists\Box a)$

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if all states are labeled by a :

this technique yields a method
to compute $Sat_{fair}(\exists\Box true)$

given: finite TS $\mathcal{T}$, atomic proposition a
CTL fairness assumption *fair*

goal: compute $Sat_{fair}(\exists\Box a)$

if all states are labeled by a :

this technique yields a method
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here: explanations only for strong fairness

$$fair = \bigwedge_{1 \leq i \leq k} (\Box\Diamond b_i \rightarrow \Box\Diamond c_i)$$

$$\textit{fair} = \bigwedge_{1 \leq i \leq k} (\Box\Diamond b_i \rightarrow \Box\Diamond c_i)$$

$$\text{fair} = \bigwedge_{1 \leq i \leq k} (\Box\Diamond b_i \rightarrow \Box\Diamond c_i)$$

$s \models_{\text{fair}} \exists\Box a$ iff there exists a path fragment

$$s_0 s_1 \dots s_n \dots s_{n+r}$$

$$fair = \bigwedge_{1 \leq i \leq k} (\Box\Diamond b_i \rightarrow \Box\Diamond c_i)$$

$s \models_{fair} \exists\Box a$ iff there exists a path fragment

$$s_0 s_1 \dots s_n \dots s_{n+r}$$

such that $r \geq 1$, $s = s_0$, $s_n = s_{n+r}$ and ...

$$\text{fair} = \bigwedge_{1 \leq i \leq k} (\Box\Diamond b_i \rightarrow \Box\Diamond c_i)$$

$s \models_{\text{fair}} \exists\Box a$ iff there exists a path fragment

$$s_0 s_1 \dots s_n \dots s_{n+r}$$

such that $r \geq 1$, $s = s_0$, $s_n = s_{n+r}$ and

- $s_j \models a$ for all $0 \leq j \leq n+r$

$$\text{fair} = \bigwedge_{1 \leq i \leq k} (\Box\Diamond b_i \rightarrow \Box\Diamond c_i)$$

$s \models_{\text{fair}} \exists\Box a$ iff there exists a path fragment

$$s_0 s_1 \dots s_n \dots s_{n+r}$$

such that $r \geq 1$, $s = s_0$, $s_n = s_{n+r}$ and

- $s_j \models a$ for all $0 \leq j \leq n+r$
- the path $s_0 s_1 \dots s_n (s_{n+1} \dots s_{n+r})^\omega$ is fair, i.e.,

$$\text{fair} = \bigwedge_{1 \leq i \leq k} (\Box\Diamond b_i \rightarrow \Box\Diamond c_i)$$

$s \models_{\text{fair}} \exists\Box a$ iff there exists a path fragment

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such that $r \geq 1$, $s = s_0$, $s_n = s_{n+r}$ and

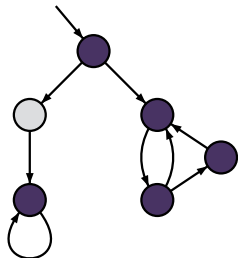
- $s_j \models a$ for all $0 \leq j \leq n+r$
- the path $s_0 s_1 \dots s_n (s_{n+1} \dots s_{n+r})^\omega$ is fair, i.e.,
for all $1 \leq i \leq k$:

$$\{s_{n+1}, \dots, s_{n+r}\} \cap \text{Sat}(b_i) = \emptyset$$
 or
$$\{s_{n+1}, \dots, s_{n+r}\} \cap \text{Sat}(c_i) \neq \emptyset$$

$\exists \Box a$ under strong fairness

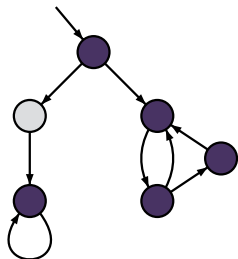
CTLFAIR4.4-19A

does $\mathcal{T} \models_{\text{fair}} \exists \Box a$ hold ?



$\bullet \models a$ $\circ \not\models a$

does $\mathcal{T} \models_{\text{fair}} \exists \Box a$ hold ?



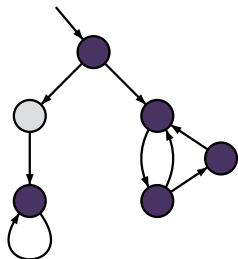
$\bullet \models a$ $\circ \not\models a$

analyze the digraph G_a that results from $\mathcal{T}$ by removing all states s with $s \not\models a$

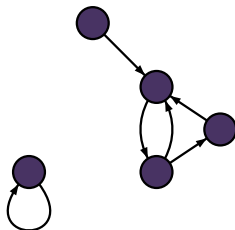
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CTLFair4.4-19A

does $\mathcal{T} \models_{\text{fair}} \exists \Box a$ hold ?



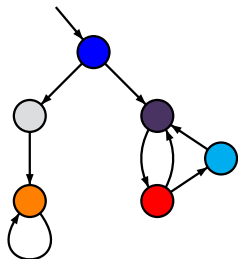
digraph G_a



$\bullet \models a$ $\circ \not\models a$

analyze the digraph G_a that results from $\mathcal{T}$ by removing all states s with $s \not\models a$

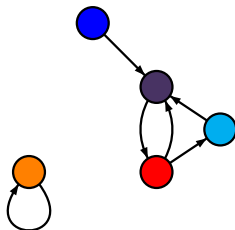
does $\mathcal{T} \models_{\text{fair}} \exists \Box a$ hold ?



$$\text{orange} \hat{=} \{b_1\} \quad \text{red} \hat{=} \{c_1\}$$

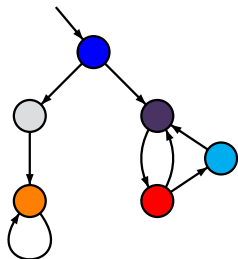
$$\text{light blue} \hat{=} \{b_2\} \quad \text{blue} \hat{=} \{c_2\}$$

digraph G_a



$$\text{fair} = (\Box \Diamond b_1 \rightarrow \Box \Diamond c_1) \wedge (\Box \Diamond b_2 \rightarrow \Box \Diamond c_2)$$

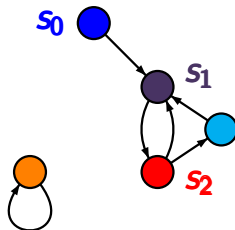
does $\mathcal{T} \models_{\text{fair}} \exists \Box a$ hold ?



$$\text{orange} \hat{=} \{b_1\} \quad \text{red} \hat{=} \{c_1\}$$

$$\text{light blue} \hat{=} \{b_2\} \quad \text{blue} \hat{=} \{c_2\}$$

digraph G_a



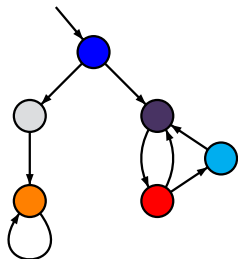
$$s_0 (s_1 s_2)^\omega \models \neg \Box \Diamond b_2 \wedge \Box \Diamond c_1$$

$$\text{fair} = (\Box \Diamond b_1 \rightarrow \Box \Diamond c_1) \wedge (\Box \Diamond b_2 \rightarrow \Box \Diamond c_2)$$

$\exists \Box a$ under strong fairness

CTLFAIR4.4-19A

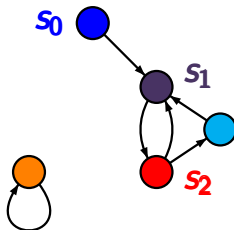
does $\mathcal{T} \models_{\text{fair}} \exists \Box a$ hold ?



$$\text{orange} \hat{=} \{b_1\} \quad \text{red} \hat{=} \{c_1\}$$

$$\text{light blue} \hat{=} \{b_2\} \quad \text{blue} \hat{=} \{c_2\}$$

digraph G_a



$$s_0 (s_1 s_2)^\omega \models \neg \Box \Diamond b_2 \wedge \Box \Diamond c_1$$

$$s_0 (s_1 s_2)^\omega \models \text{fair}$$

$$\text{fair} = (\Box \Diamond b_1 \rightarrow \Box \Diamond c_1) \wedge (\Box \Diamond b_2 \rightarrow \Box \Diamond c_2)$$

$$\text{fair} = \bigwedge_{1 \leq i \leq k} (\Box\Diamond b_i \rightarrow \Box\Diamond c_i)$$

$s \models_{\text{fair}} \exists\Box a$ iff there exists a path fragment

$$s_0 s_1 \dots s_n \dots s_{n+r}$$

such that $r \geq 1$, $s = s_0$, $s_n = s_{n+r}$ and

- $s_j \models a$ for all $0 \leq j \leq n+r$
- for all $1 \leq i \leq k$: $\{s_{n+1}, \dots, s_{n+r}\} \cap \text{Sat}(b_i) = \emptyset$
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Thus: $D = \{s_{n+1}, \dots, s_{n+r}\}$ is a strongly connected node-set of the digraph G_a

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Thus: $D = \{s_{n+1}, \dots, s_{n+r}\}$ is a strongly connected node-set of the digraph G_a (possibly not an SCC)

$$\text{fair} = \bigwedge_{1 \leq i \leq k} (\Box\Diamond b_i \rightarrow \Box\Diamond c_i)$$

$s \models_{\text{fair}} \exists\Box a$ iff there exists a non-trivial
 strongly connected node-set D of G_a such that

G_a : digraph that arises from $\mathcal{T}$ by removing all
 states s' with $s' \not\models a$

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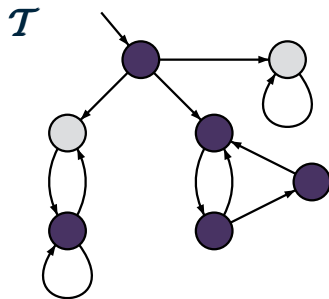
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note: if $s \models_{\text{fair}} \exists\Box a$ then there might be
 no SCC D where (1) and (2) hold

Example: computation of $Sat_{fair}(\exists \Box a)$

CTLFAIR4.4-22

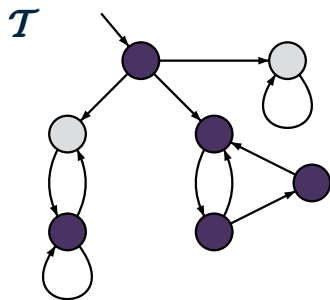


$\bullet \models a \quad \circ \not\models a$

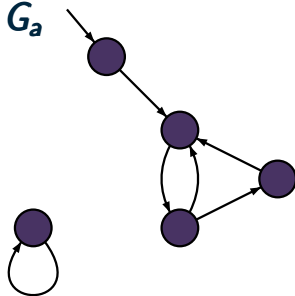
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Example: computation of $Sat_{fair}(\exists \Box a)$

CTLFair4.4-22



digraph G_a

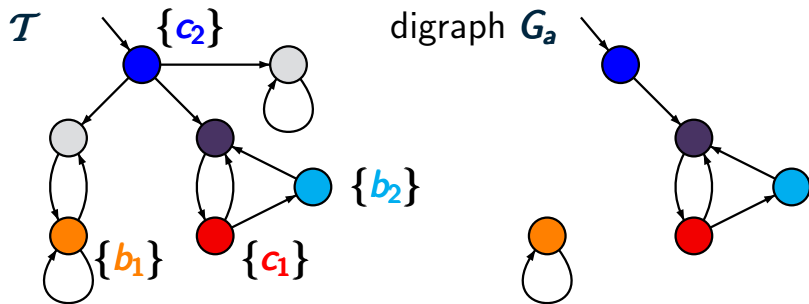


$\bullet \models a$ $\circ \not\models a$

computation of $Sat_{fair}(\exists \Box a)$
by analyzing the digraph G_a

Example: computation of $Sat_{fair}(\exists \Box a)$

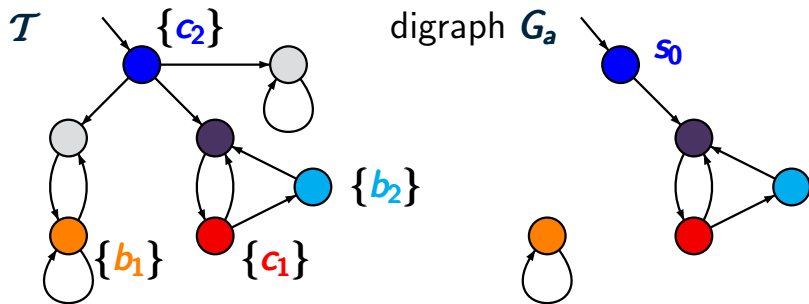
CTLFAIR4.4-22



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CTLFAIR4.4-22

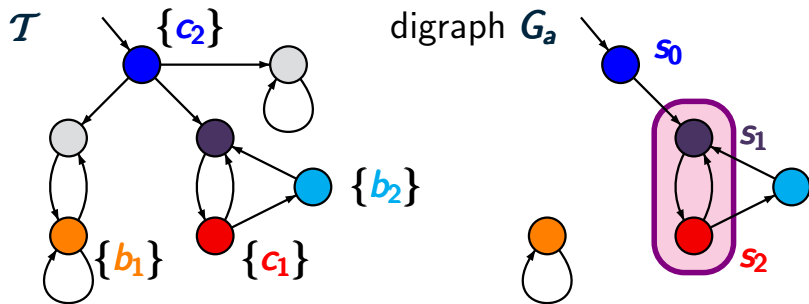


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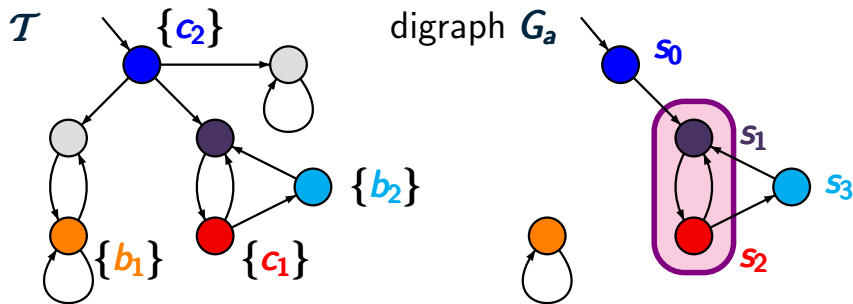


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$$s_0 \models_{fair} \exists \Box a \quad \text{as } s_0 s_1 s_2 s_1 s_2 \dots \models_{LTL} fair$$

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$$Sat_{fair}(\exists \Box a) = \{s_0, s_1, s_2, s_3\}$$

treatment of $\exists\Box$ for **CTL** with fairness

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here: explanations only for strong fairness

weak fairness and combinations of weak/strong fairness can be treated in an analogous way

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$s \models_{\text{fair}} \exists \Box a$ iff ?

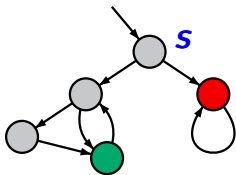
$$\text{fair} = \bigwedge_{1 \leq i \leq k} \Box \Diamond c_i$$

$s \models_{\text{fair}} \exists \Box a$ iff there exists a nontrivial SCC C in G_a that is reachable from s and $C \cap \text{Sat}(c_i) \neq \emptyset$ for $i = 1, \dots, k$

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digraph G_a



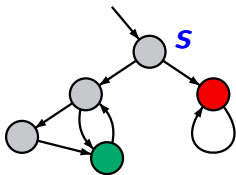
fairness assumption:

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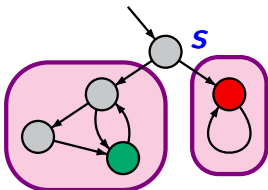
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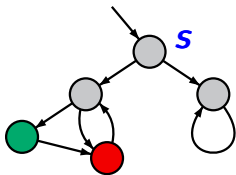
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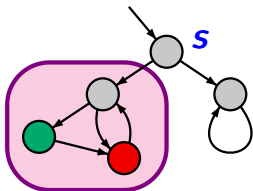
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Strong fairness: 1 fairness requirement

CTLFair4.4-25

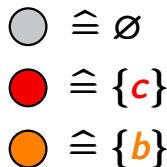
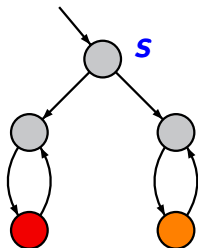
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CTLFAIR4.4-25

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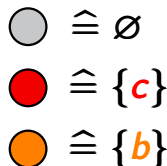
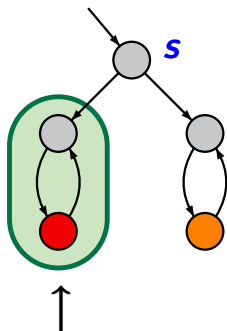


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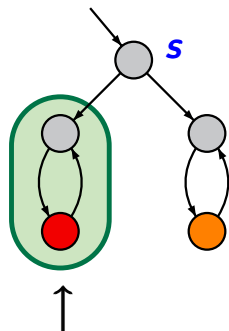
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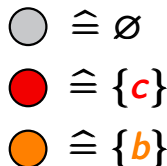
nontrivial SCC C of G_a with $C \cap \text{Sat}(c) \neq \emptyset$

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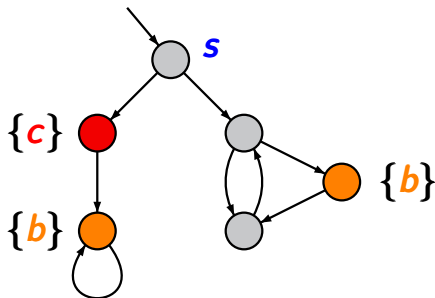
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CTLFAIR4.4-25A

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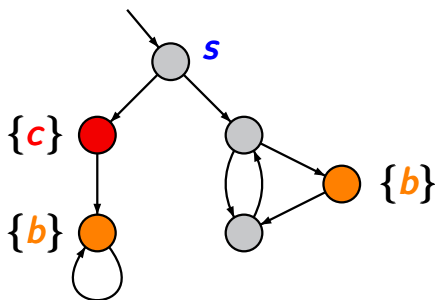


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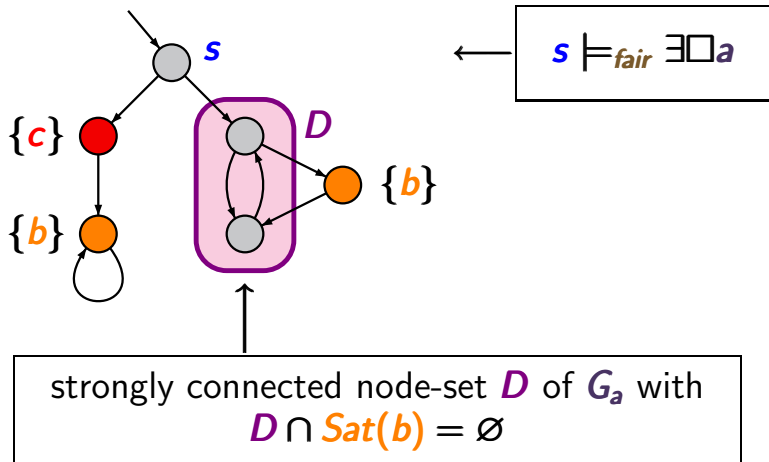
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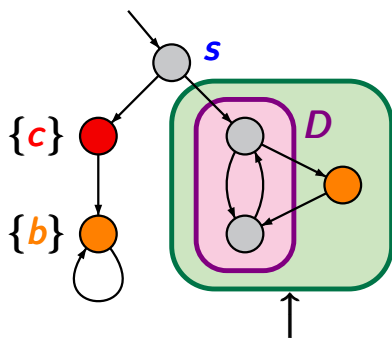


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$$s \models_{\text{fair}} \exists \Box a$$

nontrivial **SCC** C of G_a that contains a
nontrivial **SCC** D of $G_a|_c \setminus \text{Sat}(b)$

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Example: 2 strong fairness conditions

CTLFAIR4.4-26

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CTLFair4.4-26

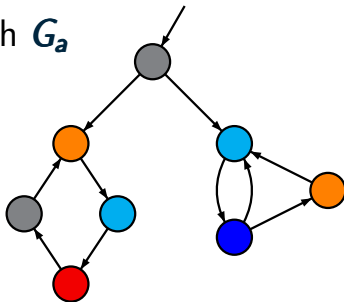
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CTLFAIR4.4-26

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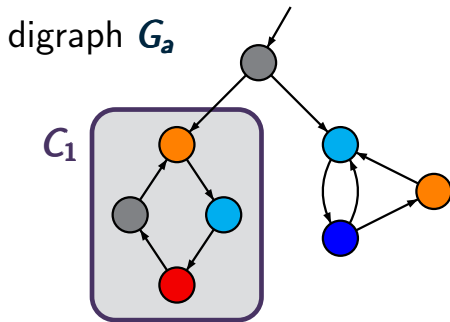
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CTLFair4.4-26

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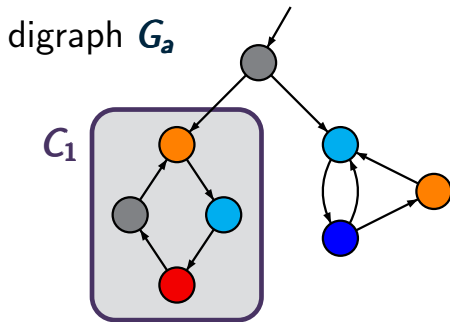


first SCC: $C_1 \cap \text{Sat}(c_2) = \emptyset$

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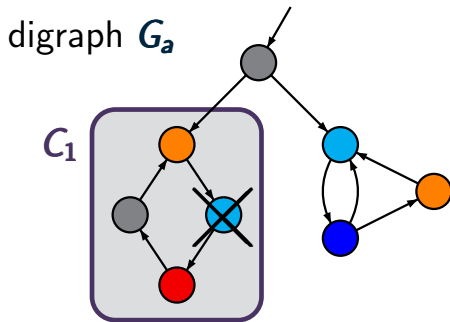
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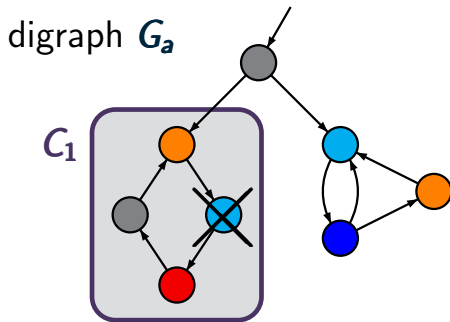
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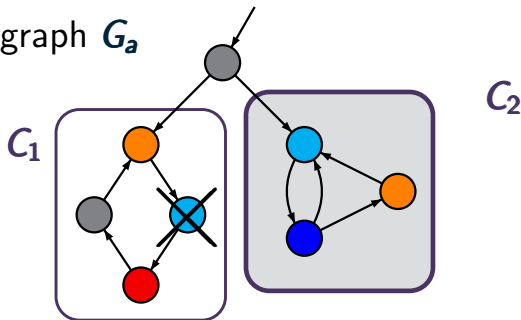
$\rightsquigarrow$ there is no cycle

Example: 2 strong fairness conditions

CTLFair4.4-26

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digraph G_a

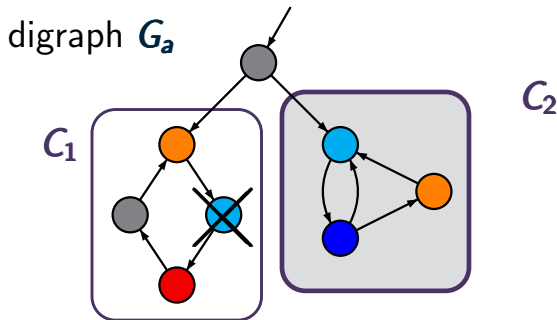


second SCC:

Example: 2 strong fairness conditions

CTLFair4.4-26

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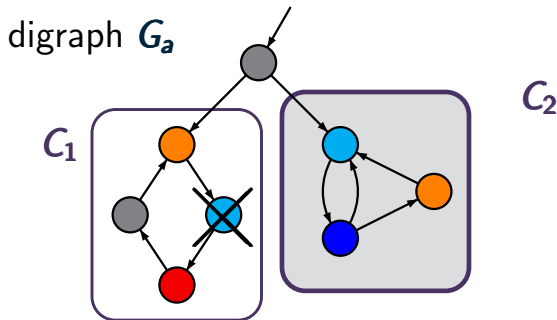


second SCC: $C_2 \cap \text{Sat}(c_1) = \emptyset$

Example: 2 strong fairness conditions

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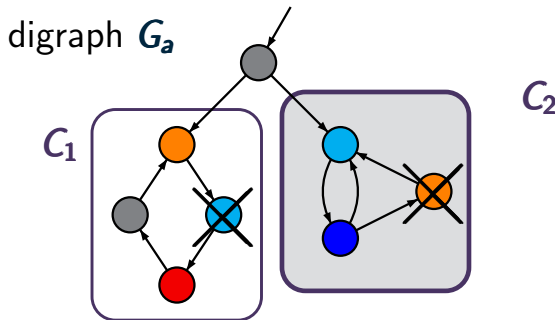
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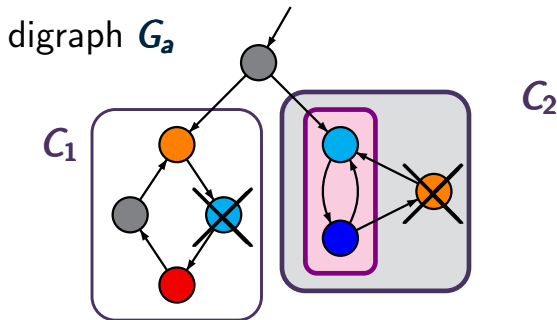
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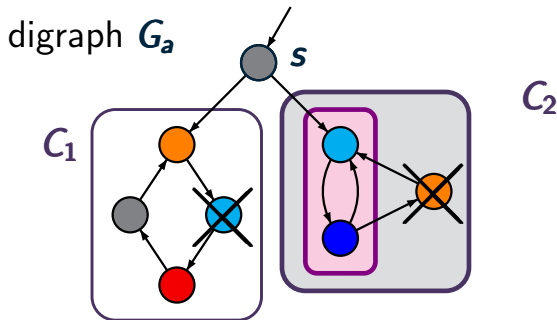
second SCC: $C_2 \cap \text{Sat}(c_1) = \emptyset$

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Example: 2 strong fairness conditions

CTLFAIR4.4-26

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hence: $s \models_{\text{fair}} \exists\Box a$

Calculation of $Sat_{fair}(\exists \square a)$

CTLFAIR4.4-27

compute the SCCs of the digraph G_a ;

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Recursive algorithm *CheckFair(...)*

CTLFAIR4.4-28

algorithm *CheckFair*($C, k, \bigwedge_{1 \leq i \leq k} (\Box \Diamond b_i \rightarrow \Box \Diamond c_i)$)

algorithm *CheckFair*($C, k, \bigwedge_{1 \leq i \leq k} (\Box \Diamond b_i \rightarrow \Box \Diamond c_i)$) returns

“true” if there exists a cyclic path fragment

$s_0 s_1 \dots s_n$ in C such that

$$(s_0 s_1 \dots s_{n-1})^\omega \models \bigwedge_{1 \leq i \leq k} (\Box \Diamond b_i \rightarrow \Box \Diamond c_i)$$

“false” otherwise

Recursive algorithm *CheckFair*(...)

CTLFair4.4-28

pseudo code for *CheckFair*(*C*, *k*, $\bigwedge_{1 \leq i \leq k} (\Box \Diamond b_i \rightarrow \Box \Diamond c_i)$)

IF $\forall i \in \{1, \dots, k\}$. *C* $\cap$ *Sat*(*c_i*) $\neq \emptyset$ THEN return “true” FI

Recursive algorithm *CheckFair*(...)

CTLFair4.4-28

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Recursive algorithm *CheckFair*(...)

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Recursive algorithm *CheckFair*(...)

CTLFair4.4-28

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OD
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Complexity of *CheckFair*(...)

CTLFAIR4.4-29

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recurrence for the time complexity:

$$T(n, k) = \dots \text{ where } n = \text{size}(C)$$

Complexity of *CheckFair*(...)

CTLFair4.4-29

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time complexity:
 $\mathcal{O}(\text{size}(C) \cdot k)$

input: finite transition system $\mathcal{T}$
 CTL fairness assumption *fair*
 CTL formula ϕ

output: “yes”, if $\mathcal{T} \models_{\text{fair}} \phi$. “no” otherwise.

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here: preprocessing

transform ϕ into an equivalent CTL formula
in **existential normal form**

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 CTL fairness assumption *fair*
 CTL formula Φ

output: “yes”, if $\mathcal{T} \models_{\text{fair}} \Phi$. “no” otherwise.

here: preprocessing

transform Φ into an equivalent CTL formula
in **existential normal form**

↑
i.e., with the basic modalities $\exists\bigcirc$, $\exists\mathbf{U}$ and $\exists\Box$

calculate $Sat_{fair}(\exists \Box true)$;

label all states in $Sat_{fair}(\exists \Box true)$ with a_{fair}

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$Sat_{fair}(\Psi) := \dots$

OD

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CASE Ψ is:

$$\begin{array}{lcl} & \vdots & \\ \exists \bigcirc a & : Sat_{fair}(\Psi) & := Sat(\exists \bigcirc (a \wedge a_{fair})); \\ \exists (a_1 \cup a_2) & : Sat_{fair}(\Psi) & := Sat(\exists (a_1 \cup (a_2 \wedge a_{fair}))); \\ \exists \Box a & : Sat_{fair}(\Psi) & := \dots \end{array}$$

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replace Ψ with a fresh atomic proposition a_Ψ

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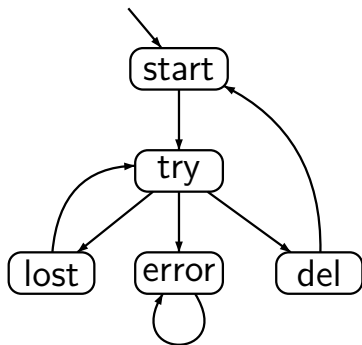
replace Ψ with a fresh atomic proposition a_Ψ

OD

IF $S_0 \subseteq Sat_{fair}(\Phi)$ THEN return “yes”

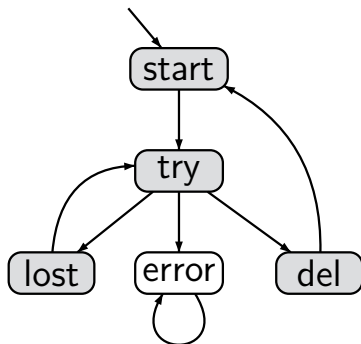
ELSE return “no”

FI



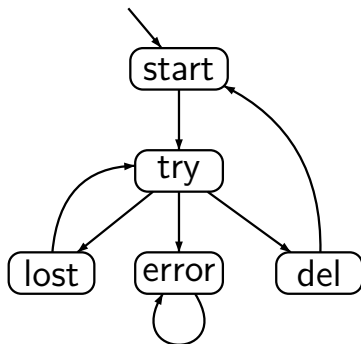
$$\Phi = \exists \Diamond \forall \bigcirc (\textit{lost} \vee \textit{del})$$

$$\textit{fair} = \Box \Diamond \exists \Diamond \textit{del}$$



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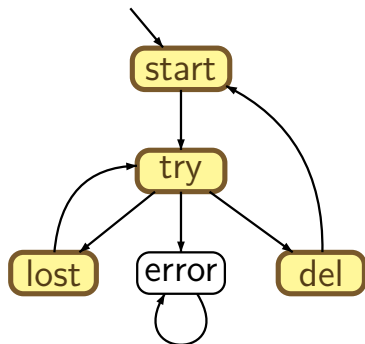
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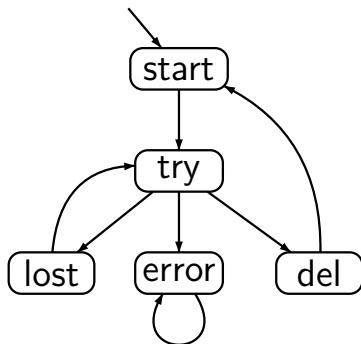
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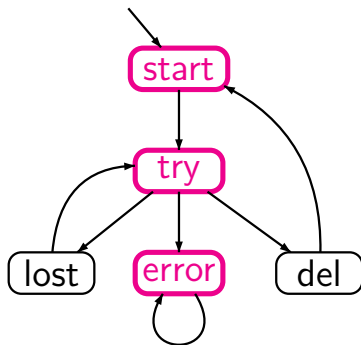
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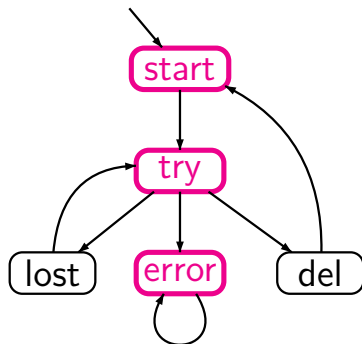


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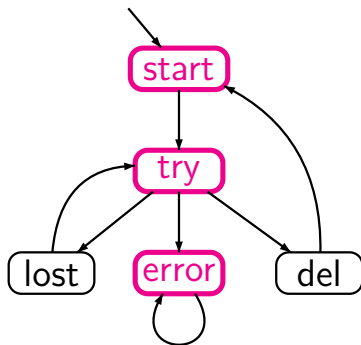
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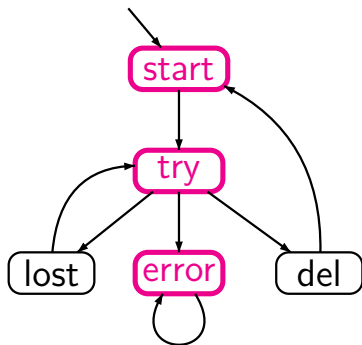
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$$Sat_{fair}(\exists \bigcirc a)$$



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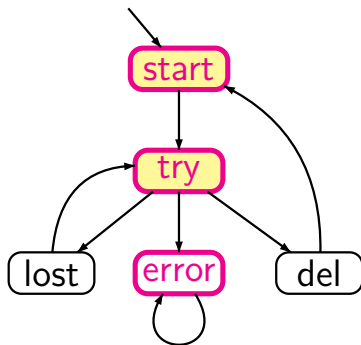
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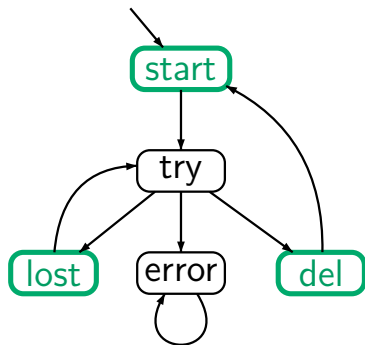
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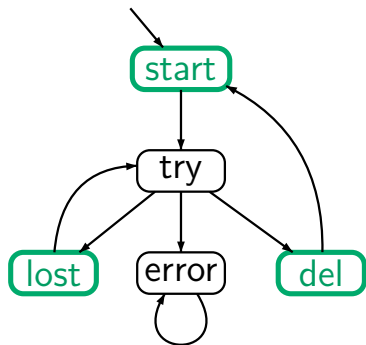
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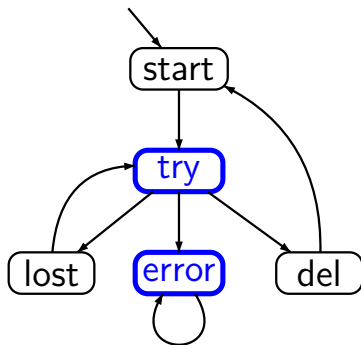
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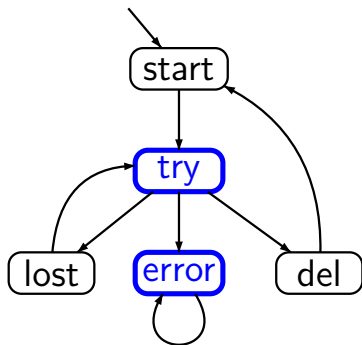
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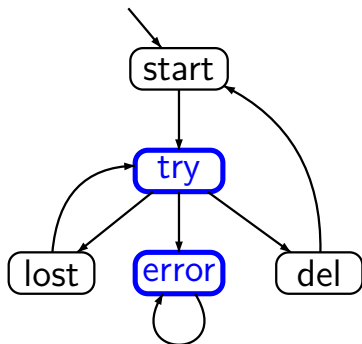
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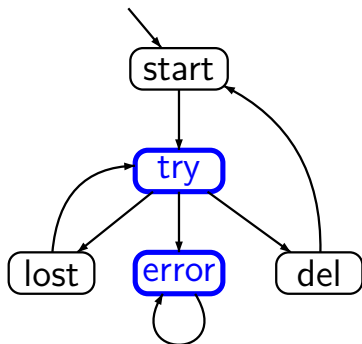
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$$Sat_{fair}(\neg \exists \bigcirc a) = \{try, error\} = Sat(b)$$

$$Sat_{fair}(\exists \Diamond b)$$



$$\Phi = \exists \Diamond \forall \bigcirc (\textit{lost} \vee \textit{del})$$

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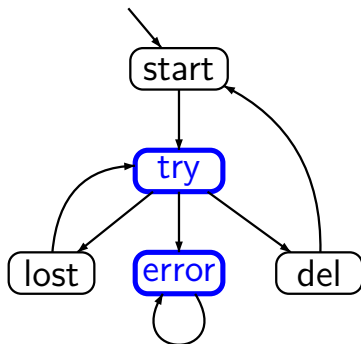
$$\rightsquigarrow \exists \Diamond \neg \exists \bigcirc \textcolor{red}{a}$$

$$\rightsquigarrow \exists \Diamond \textcolor{blue}{b}$$

$$\textit{fair} = \Box \Diamond \exists \Diamond \textit{del} \rightsquigarrow \Box \Diamond \textcolor{blue}{c} \text{ where } \textit{Sat}(\textcolor{blue}{c}) = S \setminus \{\textit{error}\}$$

$$\textit{Sat}_{\textit{fair}}(\neg \exists \bigcirc \textcolor{red}{a}) = \{\textit{try}, \textit{error}\} = \textit{Sat}(\textcolor{blue}{b})$$

$$\textit{Sat}_{\textit{fair}}(\exists \Diamond \textcolor{blue}{b}) = \textit{Sat}(\exists \Diamond (\textcolor{blue}{b} \wedge \textcolor{brown}{a}_{\textit{fair}}))$$



$$\Phi = \exists \Diamond \forall \bigcirc (lost \vee del)$$

$$\equiv \exists \Diamond \neg \exists \bigcirc (\neg lost \wedge \neg del)$$

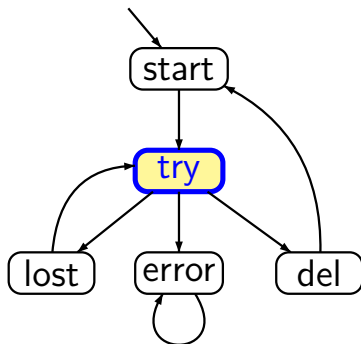
$$\rightsquigarrow \exists \Diamond \neg \exists \bigcirc a$$

$$\rightsquigarrow \exists \Diamond b$$

$$fair = \Box \Diamond \exists \Diamond del \rightsquigarrow \Box \Diamond c \text{ where } Sat(c) = S \setminus \{error\}$$

$$Sat_{fair}(\neg \exists \bigcirc a) = \{try, error\} = Sat(b)$$

$$Sat_{fair}(\exists \Diamond b) = Sat(\exists \Diamond (b \wedge a_{fair}))$$



$$\Phi = \exists \Diamond \forall \bigcirc (lost \vee del)$$

$$\equiv \exists \Diamond \neg \exists \bigcirc (\neg lost \wedge \neg del)$$

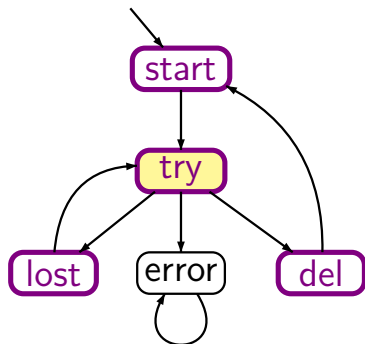
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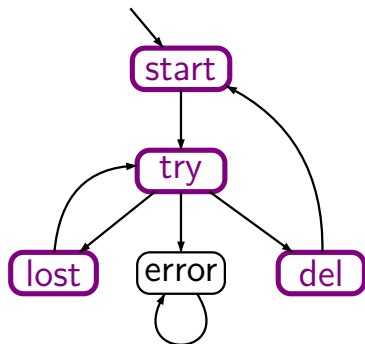
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$$\begin{aligned} Sat_{fair}(\exists \Diamond b) &= Sat(\exists \Diamond (b \wedge a_{fair})) \\ &= \{start, try, lost, del\} \end{aligned}$$



$$\Phi = \exists \Diamond \forall \bigcirc (lost \vee del)$$

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Correct or wrong?

CTLFAIR4.4-32

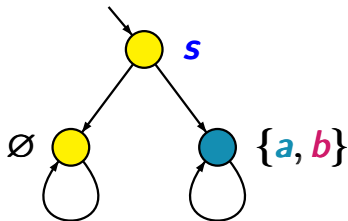
$$s \models_{\text{fair}} \forall O a \quad \text{iff} \quad s \models \forall O (a \wedge a_{\text{fair}})$$

Correct or wrong?

CTLFAIR4.4-32

$$s \models_{\text{fair}} \forall \bigcirc a \text{ iff } s \models \forall \bigcirc (a \wedge a_{\text{fair}})$$

wrong.



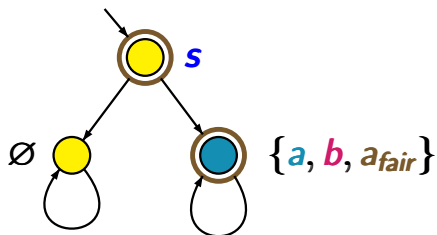
$$\text{fair} = \Box \Diamond b$$

Correct or wrong?

CTLFAIR4.4-32

$$s \models_{\text{fair}} \forall \bigcirc a \text{ iff } s \models \forall \bigcirc (a \wedge a_{\text{fair}})$$

wrong.



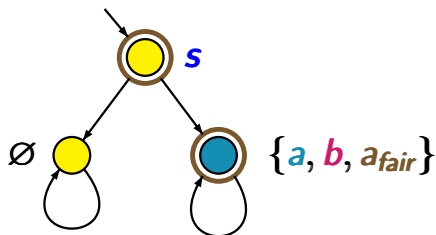
$$\text{fair} = \Box \Diamond b$$

Correct or wrong?

CTLFAIR4.4-32

$$s \models_{\text{fair}} \forall \bigcirc a \text{ iff } s \models \forall \bigcirc (a \wedge a_{\text{fair}})$$

wrong.



$$\text{fair} = \Box \Diamond b$$

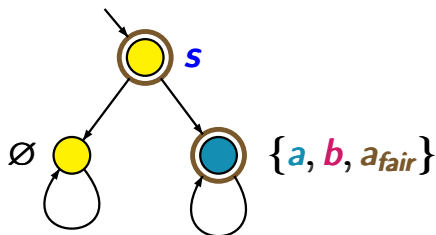
$$s \not\models \forall \bigcirc (a \wedge a_{\text{fair}})$$

Correct or wrong?

CTLFAIR4.4-32

$$s \models_{\text{fair}} \forall \bigcirc a \text{ iff } s \models \forall \bigcirc (a \wedge a_{\text{fair}})$$

wrong.



$$\text{fair} = \Box \Diamond b$$

$$s \not\models \forall \bigcirc (a \wedge a_{\text{fair}})$$

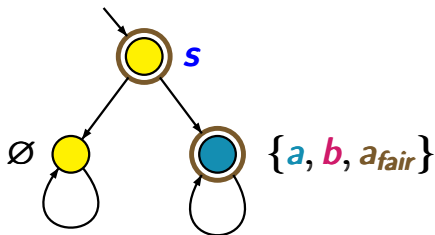
$$s \models_{\text{fair}} \forall \bigcirc a$$

Correct or wrong?

CTLFAIR4.4-32

$$s \models_{\text{fair}} \forall \bigcirc a \text{ iff } s \models \forall \bigcirc (a \wedge a_{\text{fair}})$$

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$$\text{fair} = \Box \Diamond b$$

$$s \not\models \forall \bigcirc (a \wedge a_{\text{fair}})$$

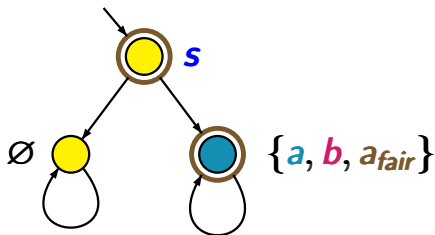
$$s \models_{\text{fair}} \forall \bigcirc a$$

but correct is:

$$s \models_{\text{fair}} \forall \bigcirc a \text{ iff } ?$$

$$s \models_{\text{fair}} \forall \bigcirc a \text{ iff } s \models \forall \bigcirc (a \wedge a_{\text{fair}})$$

wrong.



$$\text{fair} = \Box \Diamond b$$

$$s \not\models \forall \bigcirc (a \wedge a_{\text{fair}})$$

$$s \models_{\text{fair}} \forall \bigcirc a$$

but correct is:

$$s \models_{\text{fair}} \forall \bigcirc a \text{ iff } s \models \forall \bigcirc (a_{\text{fair}} \rightarrow a)$$

Correct or wrong?

CTLFAIR4.4-32B

$$s \models_{\text{fair}} \forall \Box a \quad \text{iff} \quad s \models \forall \Box (a_{\text{fair}} \rightarrow a)$$

Correct or wrong?

CTLFAIR4.4-32B

$s \models_{\text{fair}} \forall \Box a$ iff $s \models \forall \Box (a_{\text{fair}} \rightarrow a)$
iff there is no state s' reachable
from s with $s' \models \neg a \wedge a_{\text{fair}}$

Correct or wrong?

CTLFAIR4.4-32B

$s \models_{\text{fair}} \forall \Box a$ iff $s \models \forall \Box (a_{\text{fair}} \rightarrow a)$
iff there is no state s' reachable
from s with $s' \models \neg a \wedge a_{\text{fair}}$

correct

Correct or wrong?

CTLFAIR4.4-32B

$s \models_{\text{fair}} \forall \Box a$ iff $s \models \forall \Box (a_{\text{fair}} \rightarrow a)$
iff there is no state s' reachable
from s with $s' \models \neg a \wedge a_{\text{fair}}$

correct

$s \models_{\text{fair}} \forall \Box a$

$s \models_{\text{fair}} \forall \Box a$ iff $s \models \forall \Box (a_{\text{fair}} \rightarrow a)$
iff there is no state s' reachable
from s with $s' \models \neg a \wedge a_{\text{fair}}$

correct

$s \models_{\text{fair}} \forall \Box a$ iff $s \models_{\text{fair}} \neg \exists \Diamond \neg a$

$$\begin{aligned} s \models_{\text{fair}} \forall \Box a \quad \text{iff} \quad & s \models \forall \Box (a_{\text{fair}} \rightarrow a) \\ & \text{iff there is no state } s' \text{ reachable} \\ & \text{from } s \text{ with } s' \models \neg a \wedge a_{\text{fair}} \end{aligned}$$

correct

$$\begin{aligned} s \models_{\text{fair}} \forall \Box a \quad \text{iff} \quad & s \models_{\text{fair}} \neg \exists \Diamond \neg a \\ & \text{iff } s \not\models_{\text{fair}} \exists \Diamond \neg a \end{aligned}$$

$$\begin{aligned} s \models_{\text{fair}} \forall \Box a \quad \text{iff} \quad & s \models \forall \Box (a_{\text{fair}} \rightarrow a) \\ & \text{iff there is no state } s' \text{ reachable} \\ & \text{from } s \text{ with } s' \models \neg a \wedge a_{\text{fair}} \end{aligned}$$

correct

$$\begin{aligned} s \models_{\text{fair}} \forall \Box a \quad \text{iff} \quad & s \models_{\text{fair}} \neg \exists \Diamond \neg a \\ & \text{iff } s \not\models_{\text{fair}} \exists \Diamond \neg a \\ & \text{iff } s \not\models \exists \Diamond (\neg a \wedge a_{\text{fair}}) \end{aligned}$$

$$\begin{aligned} s \models_{\text{fair}} \forall \Box a \quad \text{iff} \quad & s \models \forall \Box (a_{\text{fair}} \rightarrow a) \\ & \text{iff there is no state } s' \text{ reachable} \\ & \text{from } s \text{ with } s' \models \neg a \wedge a_{\text{fair}} \end{aligned}$$

correct

$$\begin{aligned} s \models_{\text{fair}} \forall \Box a \quad \text{iff} \quad & s \models_{\text{fair}} \neg \exists \Diamond \neg a \\ & \text{iff } s \not\models_{\text{fair}} \exists \Diamond \neg a \\ & \text{iff } s \not\models \exists \Diamond (\neg a \wedge a_{\text{fair}}) \\ & \text{iff } s \models \neg \exists \Diamond (\neg a \wedge a_{\text{fair}}) \end{aligned}$$

Correct or wrong?

CTLFAIR4.4-32B

$$\begin{aligned} s \models_{\text{fair}} \forall \Box a \quad \text{iff} \quad & s \models \forall \Box (a_{\text{fair}} \rightarrow a) \\ & \text{iff there is no state } s' \text{ reachable} \\ & \text{from } s \text{ with } s' \models \neg a \wedge a_{\text{fair}} \end{aligned}$$

correct

$$\begin{aligned} s \models_{\text{fair}} \forall \Box a \quad \text{iff} \quad & s \models_{\text{fair}} \neg \exists \Diamond \neg a \\ & \text{iff } s \not\models_{\text{fair}} \exists \Diamond \neg a \\ & \text{iff } s \not\models \exists \Diamond (\neg a \wedge a_{\text{fair}}) \\ & \text{iff } s \models \neg \exists \Diamond (\neg a \wedge a_{\text{fair}}) \equiv \forall \Box (a_{\text{fair}} \rightarrow a) \end{aligned}$$

We just saw:

$$s \models_{\text{fair}} \forall \bigcirc a \quad \text{iff} \quad s \models \forall \bigcirc (a_{\text{fair}} \rightarrow a)$$

$$s \models_{\text{fair}} \forall \Box a \quad \text{iff} \quad s \models \forall \Box (a_{\text{fair}} \rightarrow a)$$

We just saw:

$$s \models_{\text{fair}} \forall \bigcirc a \quad \text{iff} \quad s \models \forall \bigcirc (a_{\text{fair}} \rightarrow a)$$

$$s \models_{\text{fair}} \forall \Box a \quad \text{iff} \quad s \models \forall \Box (a_{\text{fair}} \rightarrow a)$$

Is the following statement correct ?

$$s \models_{\text{fair}} \forall (b \cup a) \quad \text{iff} \quad s \models \forall (b \cup (a_{\text{fair}} \rightarrow a))$$

We just saw:

$$s \models_{\text{fair}} \forall \bigcirc a \quad \text{iff} \quad s \models \forall \bigcirc (a_{\text{fair}} \rightarrow a)$$

$$s \models_{\text{fair}} \forall \Box a \quad \text{iff} \quad s \models \forall \Box (a_{\text{fair}} \rightarrow a)$$

Is the following statement correct ?

$$s \models_{\text{fair}} \forall (b \cup a) \quad \text{iff} \quad s \models \forall (b \cup (a_{\text{fair}} \rightarrow a))$$

wrong.

Correct or wrong?

CTLFAIR4.4-33

$$s \models_{\text{fair}} \exists \bigcirc \exists \Diamond a \text{ iff } s \models \exists \bigcirc ((\exists \Diamond a) \wedge a_{\text{fair}})$$

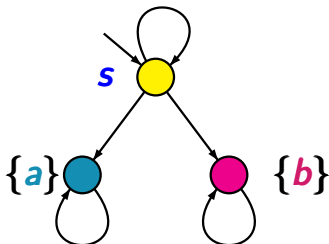
Correct or wrong?

CTLFAIR4.4-33

$$s \models_{\text{fair}} \exists \bigcirc \exists \Diamond a \quad \text{iff} \quad s \models \exists \bigcirc (\exists \Diamond a) \wedge a_{\text{fair}}$$

wrong.

$$\text{fair} = \Box \Diamond b$$



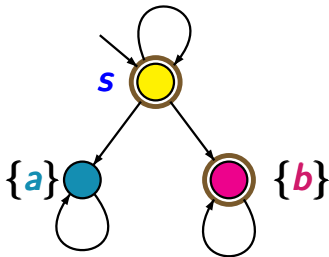
Correct or wrong?

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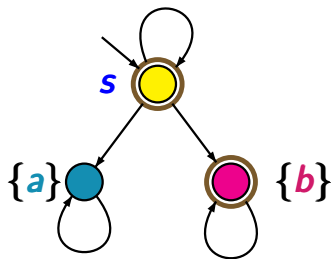


Correct or wrong?

CTLFAIR4.4-33

$$s \models_{\text{fair}} \exists \bigcirc \exists \Diamond a \quad \text{iff} \quad s \models \exists \bigcirc ((\exists \Diamond a) \wedge a_{\text{fair}})$$

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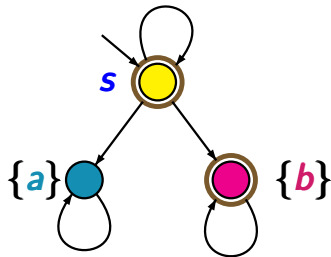
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CTLFAIR4.4-33

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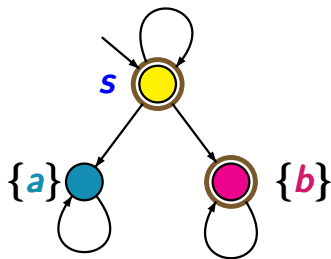
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$$s \models \exists \bigcirc ((\exists \Diamond a) \wedge a_{\text{fair}})$$

regard $s \rightarrow s$

$$s \models_{\text{fair}} \exists \bigcirc \exists \Diamond a \quad \text{iff} \quad s \models \exists \bigcirc ((\exists \Diamond a) \wedge a_{\text{fair}})$$

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$$\text{fair} = \Box \Diamond b$$

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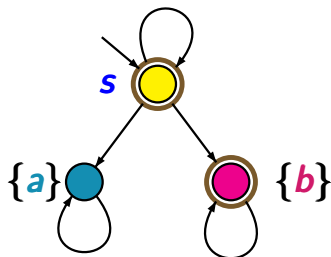
$$s \not\models_{\text{fair}} \exists \bigcirc \exists \Diamond a$$

Correct or wrong?

CTLFAIR4.4-33

$$s \models_{\text{fair}} \exists \bigcirc \exists \Diamond a \quad \text{iff} \quad s \models \exists \bigcirc ((\exists \Diamond a) \wedge a_{\text{fair}})$$

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$$\text{fair} = \Box \Diamond b$$

$$s \models \exists \bigcirc ((\exists \Diamond a) \wedge a_{\text{fair}})$$

regard $s \rightarrow s$

$$s \not\models_{\text{fair}} \exists \bigcirc \exists \Diamond a$$

(note $\text{Sat}_{\text{fair}}(\exists \Diamond a) = \emptyset$)

Correct or wrong?

CTLFAIR4.4-33

$$s \models_{\text{fair}} \exists \bigcirc \exists \Diamond a \text{ iff } s \models \exists \bigcirc ((\exists \Diamond a) \wedge a_{\text{fair}})$$

wrong.

$$s \models_{\text{fair}} \exists (a \mathbf{W} c) \text{ iff } s \models \exists (a \mathbf{W} (c \wedge a_{\text{fair}}))$$

remind: **W** = weak until

Correct or wrong?

CTLFAIR4.4-33

$$s \models_{\text{fair}} \exists \bigcirc \exists \Diamond a \quad \text{iff} \quad s \models \exists \bigcirc ((\exists \Diamond a) \wedge a_{\text{fair}})$$

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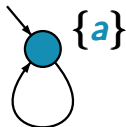
wrong.

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remind: **W** = weak until

wrong.

$$\text{fair} = \Box \Diamond b$$



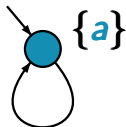
$$s \models_{\text{fair}} \exists \bigcirc \exists \Diamond a \quad \text{iff} \quad s \models \exists \bigcirc ((\exists \Diamond a) \wedge a_{\text{fair}})$$

wrong.

$$s \models_{\text{fair}} \exists (a W c) \quad \text{iff} \quad s \models \exists (a W (c \wedge a_{\text{fair}}))$$

remind: **W** = weak until

wrong.



$$\text{fair} = \Box \Diamond b$$

$$s \models \exists (a W (c \wedge a_{\text{fair}}))$$

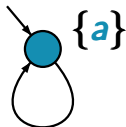
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wrong.

$$s \models_{\text{fair}} \exists (a \mathbf{W} c) \quad \text{iff} \quad s \models \exists (a \mathbf{W} (c \wedge a_{\text{fair}}))$$

remind: **W** = weak until

wrong.



$$\text{fair} = \Box \Diamond b$$

$$s \models \exists (a \mathbf{W} (c \wedge a_{\text{fair}}))$$

$$s \not\models_{\text{fair}} \exists (a \mathbf{W} c)$$

CTL fairness assumptions: formulas similar to **LTL**

$$\text{e.g., } \textit{fair} = \bigwedge_{1 \leq i \leq k} (\Box \Diamond \Psi_i \rightarrow \Box \Diamond \Phi_i)$$

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CTL satisfaction relation with fairness:

$$s \models_{\textit{fair}} \exists \varphi \quad \text{iff} \quad \text{there exists } \pi \in \textit{Paths}(s) \text{ with} \\ \pi \models_{\textit{fair}} \text{ and } \pi \models_{\textit{fair}} \varphi$$

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model checking for **CTL** with fairness:

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model checking for **CTL** with fairness:

- $\exists \bigcirc, \exists \mathbf{U}, \forall \bigcirc, \forall \Box$ via **CTL** model checker

CTL fairness assumptions: formulas similar to **LTl**

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model checking for **CTL** with fairness:

- $\exists \bigcirc, \exists \mathbf{U}, \forall \bigcirc, \forall \Box$ via **CTL** model checker
- analysis of **SCCs** for $\exists \Box, \forall \mathbf{U}$

CTL fairness assumptions: formulas similar to **LTL**

$$\text{e.g., } \textit{fair} = \bigwedge_{1 \leq i \leq k} (\Box \Diamond \Psi_i \rightarrow \Box \Diamond \Phi_i)$$

CTL satisfaction relation with fairness:

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model checking for **CTL** with fairness:

- $\exists \bigcirc, \exists \mathbf{U}, \forall \bigcirc, \forall \Box$ via **CTL** model checker
- analysis of **SCCs** for $\exists \Box, \forall \mathbf{U}$
- complexity: $\mathcal{O}(\textit{size}(\mathcal{T}) \cdot |\Phi| \cdot |\textit{fair}|)$