

Introduction to Model Checking

Winter term 2011/2012

– Series 8 –

Hand in on December 21th before the exercise class.

Exercise 1

(2 + 2 + 2 points)

We consider the release operator R which is defined as

$$\varphi R \psi := \neg(\neg \varphi U \neg \psi).$$

- (a) Informally describe the meaning of the expansion law for the release operator R . Then prove its correctness formally.
- (b) Prove the following two equivalence laws that express R by W and vice versa:
 - (I) $\varphi R \psi \equiv (\neg \varphi \wedge \psi) W (\varphi \wedge \psi)$
 - (II) $\varphi W \psi \equiv (\neg \varphi \vee \psi) R (\varphi \vee \psi)$
- (c) Prove the following equivalence law relating U and R :

$$\varphi U \psi \equiv \neg(\neg \varphi R \neg \psi)$$

Exercise 2

(1 + 1 points)

Transform the following LTL formula

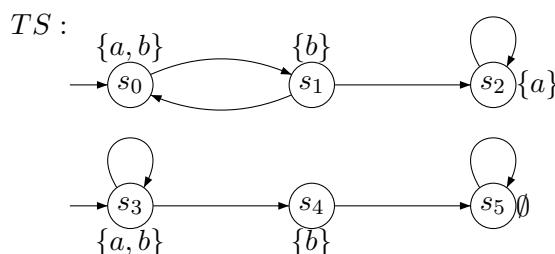
$$\varphi = \neg \diamond (\neg (a U \square c) \rightarrow (\bigcirc c \wedge a U (b \wedge \neg \square d)))$$

- (a) in positive normal form using the W operator and
- (b) in positive normal form using the R operator.

Exercise 3

(2 points)

Consider the transition system TS outlined below and the set of atomic propositions $AP = \{a, b, c\}$



and the LTL fairness assumption $fair = (\square \diamond (a \wedge b) \rightarrow \square \diamond \neg c) \wedge (\diamond \square (a \wedge b) \rightarrow \square \diamond \neg b)$.

- (a) Specify the fair paths of TS !

(b) For which of the following LTL formulas φ_i it holds $TS \models_{fair} \varphi_i$?

$$\begin{aligned}\varphi_1 &= \bigcirc \neg a \rightarrow \diamond \square a \\ \varphi_2 &= b \bigcup \square \neg b \\ \varphi_3 &= b \bigwedge \square \neg b\end{aligned}$$

In case $TS \not\models_{fair} \varphi_i$, indicate a path $\pi \in FairPaths(TS)$ for which $\pi \not\models \varphi$.