Prof. Dr. Ir. J.-P. Katoen
F. Gretz & F. Sher

Introduction to Model Checking
Winter term 2011/2012

– Series 10 –

Hand in on January 11th before the exercise class.

Exercise 1 (2 points)

Provide a non-deterministic Büchi Automaton $\mathcal{A}_i$ for each of the following LTL-formulas φ_i such that $\mathcal{L}_\omega(\mathcal{A}_i) = Words(\varphi_i)$:

(a) $\varphi_1 = \Box\big(a \vee \neg\bigcirc b\big)$

(b) $\varphi_2 = \Diamond a \vee \Box\Diamond(a \leftrightarrow b)$

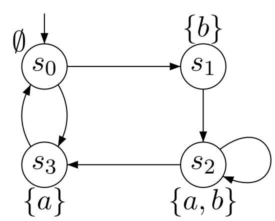
(c) $\varphi_3 = \bigcirc\bigcirc\big(a \vee \Diamond\Box b\big)$

Hint: You can infer the NBA directly, i.e. without the algorithm given in the lecture.

Exercise 2 (6 points)

We consider the LTL formula $\varphi = \Box(a \rightarrow (\neg b \mathsf{U}(a \wedge b)))$ over the set $AP = \{a, b\}$ of atomic propositions and want to check $TS \models \varphi$ wrt. the transition system outlined on the right.

TS :



a) To check $TS \models \varphi$, convert $\neg\varphi$ into an equivalent LTL-formula ψ which is constructed according to the following grammar:

$$\varphi ::= true \mid false \mid a \mid b \mid \varphi \wedge \varphi \mid \neg\varphi \mid \bigcirc\varphi \mid \varphi\mathsf{U}\varphi.$$

Then construct $closure(\psi)$.

b) Give the elementary sets wrt. $closure(\psi)$!

c) Construct the GNBA $\mathcal{G}_\psi$ by providing its initial states, its acceptance set and its transition relation. Use the algorithm given in the lecture.
Hint: It suffices to provide the transition relation as a table.

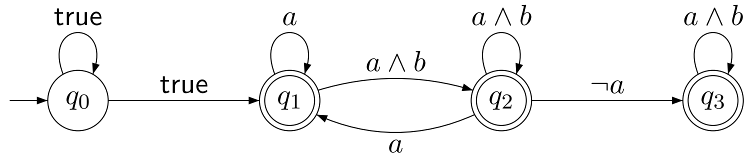
d) Now, construct an NBA $\mathcal{A}_{\neg\varphi}$ **directly** from $\neg\varphi$, i.e. without relying on $\mathcal{G}_\psi$.
Hint: Four states suffice!

e) Construct $TS \otimes \mathcal{A}_{\neg\varphi}$.

f) Use the Nested DFS algorithm from the lecture to check $TS \models \varphi$. Therefore sketch the algorithm's main steps and interpret its outcome!

Exercise 3 **(2 points)**

Consider the GNBA $\mathcal{G}$ over the alphabet $\Sigma = 2^{\{a,b\}}$ and the set $\mathcal{F} = \{\{q_1, q_3\}, \{q_2\}\}$ of acceptance sets:



a) Provide an LTL formula φ such that $\mathit{Word}(\varphi) = \mathcal{L}_\omega(\mathcal{G})$. Justify your answer!

b) Depict an NBA $\mathcal{A}$ with $\mathcal{L}_\omega(\mathcal{A}) = \mathcal{L}_\omega(\mathcal{G})$.