

## Introduction to Model Checking Winter term 2011/2012

### – Series 11 –

Hand in on January 18<sup>th</sup> before the exercise class.

#### Exercise 1

(2 points)

Let  $\varphi$  be an LTL-formula over a set of atomic propositions  $AP$ . Prove the following property:  
For all elementary sets  $B \subseteq \text{closure}(\varphi)$  and for all  $B' \in \delta(B, B \cap AP)$ , it holds:

$$\neg \bigcirc \psi \in B \iff \psi \notin B'.$$

#### Exercise 2

(4 points)

Express the following properties as CTL formulas over  $AP = \{a, b, c\}$  and provide a justification. For more complicated formulas, also comment on their subformulas!

- There exists a path on which for every state  $s$  it holds: there exists a path which starts in  $s$  and on which eventually  $a$  holds and in the next state,  $\neg a$  holds.
- There exists a state  $s$  for which it holds:  $a$  is true and on all paths starting from  $s$ ,  $c$  holds as long as  $b$  does not hold.
- On every path it holds for every state:  $a$  is valid iff ( $b$  is valid and in the previous state,  $c$  is valid).  
*Hint:* Note that this excludes the case where  $a$  is valid at the beginning.

#### Exercise 3

(2 points)

Provide two finite transition systems  $TS_1$  and  $TS_2$  (without terminal states, and over the same set of atomic propositions) and a CTL formula  $\Phi$  such that  $\text{Traces}(TS_1) = \text{Traces}(TS_2)$  and  $TS_1 \models \Phi$ , but  $TS_2 \not\models \Phi$ .

#### Exercise 4

(1 + 1 points)

Prove or disprove the following implications:

- Let  $\Phi_1 = \forall \Diamond a \vee \forall \Diamond b$  and  $\Phi_2 = \forall \Diamond (a \vee b)$ .  
Prove or disprove the following implications:  $\Phi_1 \implies \Phi_2$  and  $\Phi_2 \implies \Phi_1$ .
- Now consider  $\Psi_1 = \exists (a \cup \exists (b \cup c))$  and  $\Psi_2 = \exists (\exists (a \cup b) \cup c)$ .  
Again, prove or disprove  $\Psi_1 \implies \Psi_2$  and  $\Psi_2 \implies \Psi_1$ .