RHEINISCH-WESTFÄLISCHE TECHNISCHE HOCHSCHULE AACHEN

LEHRSTUHL FÜR INFORMATIK 2
RWTH Aachen · D-52056 Aachen · GERMANY
http://www-i2.informatik.rwth-aachen.de/

Prof. Dr. Ir. J.-P. Katoen
F. Gretz & K. v. d. Pol

Introduction to Model Checking
Winter term 2013/2014

– Series 7 –

Hand in on December 11th before the exercise class or in the box in front of the chair's secretary's office.

Exercise 1 (2 points)

a) Give an LT property that is not an ω-regular property.

b) Give an ω-regular property that is not expressible in LTL.

Exercise 2 (3 points)

Let φ and ψ be LTL formulae. Consider the following new operators:

a) "At next" $\varphi \,\mathsf{AX}\, \psi$:

$$A_1 A_2 \ldots \models \varphi \,\mathsf{AX}\, \psi \iff \text{for all } i \geq 0 \text{ where } A_i A_{i+1} \ldots \models \psi, \text{ for which there exists no } 0 \leq j < i \text{ where } A_j A_{j+1} \ldots \models \psi, \; A_i A_{i+1} \ldots \models \varphi \text{ holds}$$

b) "While" $\varphi \,\mathsf{WH}\, \psi$:

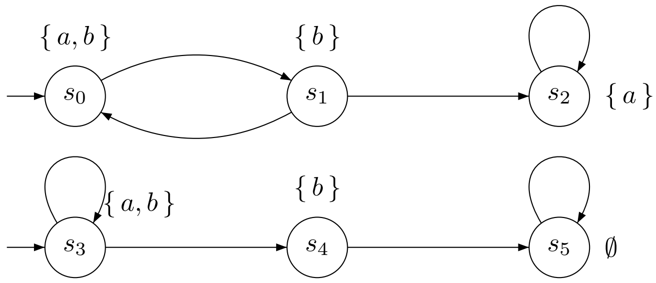
$$A_1 A_2 \ldots \models \varphi \,\mathsf{WH}\, \psi \iff \text{for all } i \geq 0 \text{ where } A_j A_{j+1} \ldots \models \psi \text{ for all } 0 \leq j < i, \; A_k A_{k+1} \ldots \models \varphi \text{ for all } 0 \leq k < i$$

c) "Before" $\varphi \,\mathsf{B}\, \psi$:

$$A_1 A_2 \ldots \models \varphi \,\mathsf{B}\, \psi \iff \text{for all } i \geq 0 \text{ where } A_i A_{i+1} \ldots \models \psi, \text{ there exists some } 0 \leq j < i \text{ where } A_j A_{j+1} \ldots \models \varphi$$

Show that these operators are LTL-definable by providing equivalent LTL formulae. You may use both the until and weak until operator.

Exercise 3 **(4 points)**



Consider the transition system T above with the set $AP = \{ a, b, c \}$ of atomic propositions. Note that this is a single transition system with two initial states. Consider the LTL fairness assumption

$$fair \;=\; \big(\Box\Diamond(a \;\&\; b) \;\rightarrow\; \Box\Diamond\neg c\big) \;\;\wedge\;\; \big(\Diamond\Box(a \;\&\; b) \rightarrow \Box\Diamond\neg b\big).$$

Questions:

a) Determine the fair paths in T, i.e., the initial, infinite paths satisfying *fair*

b) For each of the following LTL formulae:

$$\begin{aligned} \varphi_1 &= b \,\mathsf{U}\, \Box\neg b \\ \varphi_2 &= b \,\mathsf{W}\, \Box\neg b \\ \varphi_3 &= \bigcirc \bigcirc\, b \,\mathsf{U}\, \Box\neg b \end{aligned}$$

determine whether $\mathrm{T} \models_{fair} \varphi_i$. In case $\mathrm{T} \not\models_{fair} \varphi_i$, indicate a path $\pi \in Paths(\mathrm{T})$ for which $\pi \not\models \varphi_i$.

(c) Redo the previous task but ignore the fairness assumption.

Exercise 4 **(1 points)**

We consider the release operator R which is defined by

$$\varphi \,\mathsf{R}\, \psi \stackrel{\text{def}}{=} \neg(\neg\varphi \,\mathsf{U}\, \neg\psi) \;\; .$$

Prove the expansion law

$$\varphi_1 \,\mathsf{R}\, \varphi_2 \;\equiv\; \varphi_2 \wedge (\varphi_1 \vee \bigcirc(\varphi_1 \,\mathsf{R}\, \varphi_2)) \;\; .$$