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LEHRSTUHL FÜR INFORMATIK 2
RWTH Aachen · D-52056 Aachen · GERMANY
http://www-i2.informatik.rwth-aachen.de/

Prof. Dr. Ir. J.-P. Katoen
F. Gretz & K. v. d. Pol

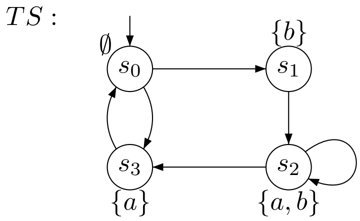
Introduction to Model Checking
Winter term 2013/2014

– Series 10 –

Hand in on January 15th before the exercise class or in the box in front of the chair's secretary's office.

Exercise 1 (6 points)

We consider the LTL formula $\varphi = \Box(a \to (\neg b \,\mathsf{U}\, (a \wedge b)))$ over the set $AP = \{a, b\}$ of atomic propositions and want to check $TS \models \varphi$ wrt. the transition system outlined on the right.



a) To check $TS \models \varphi$, convert $\neg\varphi$ into an equivalent LTL-formula ψ which is constructed according to the following grammar:

$$\varphi ::= \mathit{true} \mid \mathit{false} \mid a \mid b \mid \varphi \wedge \varphi \mid \neg\varphi \mid \bigcirc \varphi \mid \varphi \,\mathsf{U}\, \varphi.$$

Then construct $\mathit{closure}(\psi)$.

b) Give the elementary sets wrt. $\mathit{closure}(\psi)$!

c) Construct the GNBA $\mathcal{G}_\psi$ by providing its initial states, its acceptance set and its transition relation. Use the algorithm given in the lecture.
Hint: It suffices to provide the transition relation as a table.

d) Now, construct an NBA $\mathcal{A}_{\neg\varphi}$ **directly** from $\neg\varphi$, i.e. without relying on $\mathcal{G}_\psi$.
Hint: Four states suffice!

e) Construct $TS \otimes \mathcal{A}_{\neg\varphi}$.

f) Use the Nested DFS algorithm from the lecture to check $TS \models \varphi$. Therefore sketch the algorithm's main steps and interpret its outcome!

Exercise 2 (2 points)

Here we consider a **non-probabilistic** version of the famous Monty Hall game. Consider the following description of a play:

- There are three closed doors: A, B, and C. Behind exactly one door is the grand prize: a car.
- The contestant chooses a door.
- The game show host, who knows what is behind each door, opens a door that the contestant did *not* choose, revealing a goat. Note that there is always at least one such door that the game show host can open without revealing the car.
- The contestant chooses to either stay with his decision, or to switch to the other unopened door.
- The door of the contestant's *second* decision is opened and the contestant either wins or loses.

(a) Model this process as a transition system. Assume the car is behind gate one[1]! Explain the atomic propositions you used.

(b) Give a CTL-formula expressing the following property: "after the game show host opens the door with the goat, the contestant can still choose the door that will make him win and he can still choose the door that will make him lose".

Exercise 3 **(2 points)**

Prove or disprove the following implications:

(a) Let $\Phi_1 = \forall\Diamond\, a \vee \forall\Diamond\, b$ and $\Phi_2 = \forall\Diamond\,(a \vee b)$.
Prove or disprove the following implications: $\Phi_1 \implies \Phi_2$ and $\Phi_2 \implies \Phi_1$.

(b) Now consider $\Psi_1 = \exists(a \,\mathsf{U}\, \exists(b \,\mathsf{U}\, c))$ and $\Psi_2 = \exists(\exists(a \,\mathsf{U}\, b) \,\mathsf{U}\, c)$.
Again, prove or disprove $\Psi_1 \implies \Psi_2$ and $\Psi_2 \implies \Psi_1$.

[1] The other two possibilities are symmetric and can therefore be skipped.