

# Modeling Concurrent and Probabilistic Systems

Winter Term 07/08

## — Series 8 —

Hand in until December 14 before the exercise class.

### Exercise 1

(1+3 points)

Consider the following  $\pi$ -calculus process

$$P = \text{new } z((\bar{x}\langle y \rangle + z(w).\bar{w}\langle y \rangle) \parallel x(u).\bar{u}\langle v \rangle \parallel \bar{x}\langle z \rangle).$$

- a) Prove that  $P \equiv x(u).\bar{u}\langle v \rangle \parallel \text{new } z((\bar{x}\langle y \rangle + z(w).\bar{w}\langle y \rangle) \parallel \bar{x}\langle z \rangle)$ .
- b) Formally derive all successors of  $P$  using the reaction rules of the  $\pi$ -calculus.

### Exercise 2

(2 points)

Prove that if  $x$  is not free in  $Q$  then  $\text{new } x Q \equiv Q$ .

### Exercise 3

(2 points)

By using structural congruence, exhibit the redex in

$$x(z).\bar{y}\langle z \rangle \parallel !(\text{new } y (\bar{x}\langle y \rangle .Q))$$

and formally derive the result of the reaction!

### Exercise 4

(4 points)

We wish to send messages consisting of more than one name. So we want to allow the forms

$$x(y_1, \dots, y_n).P \text{ and } \bar{x}\langle z_1, \dots, z_n \rangle.Q$$

(where all the  $y_i$  are distinct) for any  $n \geq 0$ . For a correct encoding, we have to ensure that there cannot be an inference on the channel along which a composite message is sent. To send a message  $\langle z_1, \dots, z_n \rangle$ , we first send a *fresh* name  $w$  along  $x$ , then send the components  $z_i$  one by one along  $w$ . So we translate the multiple action prefixes as follows:

$$\begin{aligned} x(y_1, \dots, y_n).P &\mapsto x(w).w(y_1).\dots.w(y_n).P \\ \bar{x}\langle z_1, \dots, z_n \rangle.Q &\mapsto \text{new } w (\bar{x}\langle w \rangle.\bar{w}\langle z_1 \rangle.\dots.\bar{w}\langle z_n \rangle.Q) \end{aligned} \quad \text{where } w \notin \text{fn}(Q)$$

Apply this encoding to

$$x(y_1 y_2).P \parallel \bar{x}\langle z_1 z_2 \rangle.Q \parallel \bar{x}\langle z'_1 z'_2 \rangle.Q'.$$

Do at least two reduction sequences to convince yourself that only the right replacements occur!