Modeling Concurrent and Probabilistic Systems

Lecture 7: Strong Simulation and Weak Bisimulation

Joost-Pieter Katoen Thomas Noll

Software Modeling and Verification Group
RWTH Aachen University
noll@cs.rwth-aachen.de

http://www-i2.informatik.rwth-aachen.de/i2/mcps07/

Winter Semester 2007/08

Outline

1. Strong Simulation
2. Definition of Weak Bisimulation
3. Properties of Weak Bisimulation

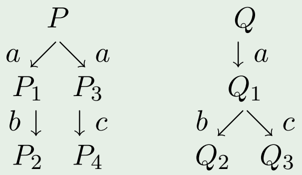
Observation: sometimes, the concept of strong bisimulation is too strong (example: extending a system by new features)

Definition 7.1 (Strong simulation)

A relation $\rho \subseteq Prc \times Prc$ is called a strong simulation if, whenever $P \rho Q$ and $P \xrightarrow{\alpha} P'$, there exists $Q' \in Prc$ such that $Q \xrightarrow{\alpha} Q'$ and $P' \rho Q'$.
We say that Q strongly simulates P if there exists a strong simulation ρ such that $P \rho Q$.

Thus: if Q strongly simulates P, then whatever transition path P takes, Q can match it by a path which retains all of P's options.

Example 7.2



Q strongly simulates P, but not vice versa

Corollary 7.3

If $P \sim Q$, then Q strongly simulates P, and P strongly simulates Q.

Proof.

A strong bisimulation $\rho \subseteq Prc \times Prc$ for $P \sim Q$ is a strong simulation for both directions. □

Caveat: the converse does generally not hold!

Example 7.4

$$\begin{array}{ccc} & P & \\ a \swarrow & & \searrow a \\ P_1 & & P_3 \\ b \downarrow & & \\ P_2 & & \end{array} \qquad \begin{array}{c} Q \\ \downarrow a \\ Q_1 \\ \downarrow b \\ Q_2 \end{array}$$

Q simulates P and vice versa, but $P \not\sim Q$

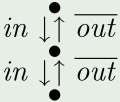
Outline

Example 7.5

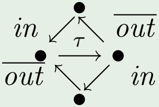
Sequential and parallel two-place buffer:

$$\begin{aligned} B_0(in, out) &= in.B_1(in, out) \\ B_1(in, out) &= \overline{out}.B_0(in, out) + \\ &\quad\; in.B_2(in, out) \\ B_2(in, out) &= \overline{out}.B_1(in, out) \end{aligned}$$

$$\begin{aligned} B_{\|}(in, out) &= \mathsf{new}\ com\ (B(in, com) \parallel \\ &\qquad\qquad B(com, out)) \\ B(in, out) &= in.\overline{out}.B(in, out) \end{aligned}$$



$\not\sim$



Definition of Weak Bisimulation I

Idea: abstract from silent actions

Definition 7.6

- Given $w \in Act^*$, $\hat{w} \in (N \cup \overline{N})^*$ denotes the sequence of non-τ-actions in w (in particular, $\hat{\tau^n} = \varepsilon$ for every $n \in \mathbb{N}$).
- For $w = \alpha_1 \ldots \alpha_n \in Act^*$ and $P, Q \in Prc$, we let

$$P \overset{w}{\Longrightarrow} Q \iff P \, (\overset{\tau}{\longrightarrow})^* \overset{\alpha_1}{\longrightarrow} (\overset{\tau}{\longrightarrow})^* \ldots (\overset{\tau}{\longrightarrow})^* \overset{\alpha_n}{\longrightarrow} (\overset{\tau}{\longrightarrow})^* \, Q$$

  (and hence: $\overset{\varepsilon}{\Longrightarrow} \; = \; (\overset{\tau}{\longrightarrow})^*$).
- A relation $\rho \subseteq Prc \times Prc$ is called a weak bisimulation if $P \rho Q$ implies, for every $\alpha \in Act$,
  1. $P \overset{\alpha}{\longrightarrow} P' \implies$ ex. $Q' \in Prc$ such that $Q \overset{\hat{\alpha}}{\Longrightarrow} Q'$ and $P' \rho Q'$
  2. $Q \overset{\alpha}{\longrightarrow} Q' \implies$ ex. $P' \in Prc$ such that $P \overset{\hat{\alpha}}{\Longrightarrow} P'$ and $P' \rho Q'$
- $P, Q \in Prc$ are called weakly bisimilar (notation: $P \approx Q$) if there exists a weak bisimulation ρ such that $P \rho Q$.

Remark: each of the two clauses in the definition of weak bisimulation subsumes two cases:

- $P \xrightarrow{\alpha} P'$ where $\alpha \neq \tau$
  $\implies$ ex. $Q' \in \mathit{Prc}$ such that $Q \, (\xrightarrow{\tau})^* \xrightarrow{\alpha} (\xrightarrow{\tau})^* \, Q'$ and $P' \rho Q'$
- $P \xrightarrow{\tau} P'$
  $\implies$ ex. $Q' \in \mathit{Prc}$ such that $Q \, (\xrightarrow{\tau})^* \, Q'$ and $P' \rho Q'$
  (where $Q' = Q$ is admissible)

Example 7.7

on the board

Outline

Properties of Weak Bisimulation I

Theorem 7.8

$\approx$ *is an equivalence relation.*

Proof.

in analogy to the corresponding proof for $\sim$ (Theorem 4.2)

In particular, the following characterization is still valid:

$$\approx = \bigcup \{\rho \mid \rho \text{ weak bisimulation}\},$$

i.e., $\approx$ is again itself a weak bisimulation. □

Properties of Weak Bisimulation II

Moreover Definition 7.6 implies that every strong bisimulation is also a weak one (since, for every $\alpha \in Act$, $\xrightarrow{\alpha} \subseteq \xRightarrow{\hat{\alpha}}$). This yields the desired connection to LTS equivalence: for every $P, Q \in Prc$,

$$LTS(P) = LTS(Q) \implies P \sim Q \implies P \approx Q.$$

Furthermore trace equivalence is implied if the definition is adapted:

$$P \approx Q \implies \hat{Tr}(P) = \hat{Tr}(Q)$$

where $\hat{Tr}(P) := \{\hat{w} \mid w \in Tr(P)\} \subseteq (N \cup \overline{N})^*$.

Properties of Weak Bisimulation III

Another important property is

Lemma 7.9

For every $P \in Prc$,

$$P \approx \tau.P$$

Proof.

We show that

$$\rho := \{(P, \tau.P)\} \cup id_{Prc}$$

is a weak bisimulation:

1. let $P \xrightarrow{\alpha} P'$
   $\Longrightarrow\ \tau.P \xrightarrow{\tau} P \xrightarrow{\alpha} P'$
   $\Longrightarrow\ \tau.P \stackrel{\hat{\alpha}}{\Longrightarrow} P'$ with $P' \rho P'$ (since $id_{Prc} \subseteq \rho$)
2. the only transition of $\tau.P$ is $\tau.P \xrightarrow{\tau} P$;
   it is simulated by $P \stackrel{\varepsilon}{\Longrightarrow} P$ with $P \rho P$

□

Properties of Weak Bisimulation IV

Using Lemma 7.9, however, we can show that $\approx$ is not a congruence:

It is true that	$b.\mathsf{nil} \approx \tau.b.\mathsf{nil}$	(Theorem 7.8, Lemma 7.9)
but	$a.\mathsf{nil} + b.\mathsf{nil} \not\approx a.\mathsf{nil} + \tau.b.\mathsf{nil}$	(Example 7.7(b))

The other operators are uncritical, i.e., weak bisimilarity is preserved under prefixing, parallel composition, and restriction.

Properties of Weak Bisimulation V

Also deadlock sensitivity is guaranteed if τ-actions are appropriately handled:

Theorem 7.10

Let $P, Q \in Prc$ such that $P \approx Q$. Then, for every $w \in (N \cup \overline{N})^*$,

$$P \stackrel{w}{\Longrightarrow}\not\longrightarrow \quad \iff \quad Q \stackrel{w}{\Longrightarrow}\not\longrightarrow .$$

Proof.

analogously to Theorem 5.4 (induction on $|w|$) □