Modeling Concurrent and Probabilistic Systems
Lecture 8: Observation Congruence

Joost-Pieter Katoen   Thomas Noll

Software Modeling and Verification Group
RWTH Aachen University
noll@cs.rwth-aachen.de

http://www-i2.informatik.rwth-aachen.de/i2/mcps07/

Winter Semester 2007/08

Outline

Repetition: Definition of Weak Bisimulation

Definition

- Given $w \in Act^*$, $\hat{w} \in (N \cup \overline{N})^*$ denotes the sequence of non-τ-actions in w (in particular, $\hat{\tau^n} = \varepsilon$ for every $n \in \mathbb{N}$).

- For $w = \alpha_1 \ldots \alpha_n \in Act^*$ and $P, Q \in Prc$, we let

$$P \stackrel{w}{\Longrightarrow} Q \iff P \; (\stackrel{\tau}{\longrightarrow})^* \stackrel{\alpha_1}{\longrightarrow} (\stackrel{\tau}{\longrightarrow})^* \ldots (\stackrel{\tau}{\longrightarrow})^* \stackrel{\alpha_n}{\longrightarrow} (\stackrel{\tau}{\longrightarrow})^* \; Q$$

(and hence: $\stackrel{\varepsilon}{\Longrightarrow} \; = \; (\stackrel{\tau}{\longrightarrow})^*$).

- A relation $\rho \subseteq Prc \times Prc$ is called a weak bisimulation if $P\rho Q$ implies, for every $\alpha \in Act$,

  1. $P \stackrel{\alpha}{\longrightarrow} P' \implies$ ex. $Q' \in Prc$ such that $Q \stackrel{\hat{\alpha}}{\Longrightarrow} Q'$ and $P'\rho Q'$
  2. $Q \stackrel{\alpha}{\longrightarrow} Q' \implies$ ex. $P' \in Prc$ such that $P \stackrel{\hat{\alpha}}{\Longrightarrow} P'$ and $P'\rho Q'$

- $P, Q \in Prc$ are called weakly bisimilar (notation: $P \approx Q$) if there exists a weak bisimulation ρ such that $P\rho Q$.

Properties

1. $P \sim Q \implies P \approx Q$

2. $\approx$ is an equivalence relation

3. $LTS(P) = LTS(Q) \implies P \approx Q$

4. $P \approx Q \implies \hat{Tr}(P) = \hat{Tr}(Q)$

5. $\approx$ is (non-τ) deadlock sensitive

6. For every $P \in Prc$, $P \approx \tau.P$

7. $\approx$ is not a congruence:

   It is true that $b.\mathsf{nil} \approx \tau.b.\mathsf{nil}$
   but $\qquad a.\mathsf{nil} + b.\mathsf{nil} \not\approx a.\mathsf{nil} + \tau.b.\mathsf{nil}$

Outline

Lemma 8.1

For every $P, Q, R \in Prc$,

1. $P + Q \approx Q + P$
2. $P + (Q + R) \approx (P + Q) + R$
3. $P + \mathsf{nil} \approx P$
4. $P \parallel Q \approx Q \parallel P$
5. $P \parallel (Q \parallel R) \approx (P \parallel Q) \parallel R$
6. $P \parallel \mathsf{nil} \approx P$

Proof.

similar to Lemma 5.2 (strong bisimulation; omitted) □

Outline

Definition of Observation Congruence I

Goal: introduce an equivalence which has most of the desirable properties of $\approx$ and which is preserved under all CCS operators
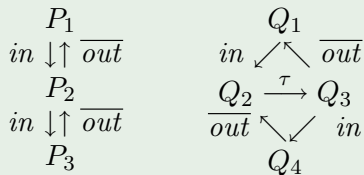

Definition 8.2

$P, Q \in Prc$ are called observationally congruent (notation: $P \simeq Q$) if, for every $\alpha \in Act$,

1. $P \xrightarrow{\alpha} P' \implies$ ex. $Q' \in Prc$ such that $Q \xLongrightarrow{\alpha} Q'$ and $P' \approx Q'$
2. $Q \xrightarrow{\alpha} Q' \implies$ ex. $P' \in Prc$ such that $P \xLongrightarrow{\alpha} P'$ and $P' \approx Q'$

Remark: $\simeq$ differs from $\approx$ only in the use of $\xLongrightarrow{\alpha}$ rather than $\xLongrightarrow{\hat{\alpha}}$, i.e., it requires τ-actions from P or Q to be simulated by at least one τ-step in the other process. This only applies to the first step; the successors just have to satisfy $P' \approx Q'$ (and not $P' \simeq Q'$).

Definition of Observation Congruence II

Example 8.3

1. Sequential and parallel two-place buffer:

$$
\begin{array}{cc}
\begin{array}{c}
P_1 \\
in \downarrow\uparrow \overline{out} \\
P_2 \\
in \downarrow\uparrow \overline{out} \\
P_3
\end{array}
&
\begin{array}{c}
Q_1 \\
in \swarrow \nwarrow \overline{out} \\
Q_2 \xrightarrow{\tau} Q_3 \\
\overline{out} \nwarrow \swarrow \; in \\
Q_4
\end{array}
\end{array}
$$

   $P_1 \simeq Q_1$ since $P_1 \approx Q_1$ (cf. Example 7.7) and neither P_1 nor Q_1 has initial τ-steps

2. $\tau.a.\mathsf{nil} \not\simeq a.\mathsf{nil}$
   (since $\quad \tau.a.\mathsf{nil} \xrightarrow{\tau} \quad$ but $\quad a.\mathsf{nil} \not\xrightarrow{\tau}$)

3. $a.\tau.\mathsf{nil} \simeq a.\mathsf{nil}$
   (since $\tau.\mathsf{nil} \approx \mathsf{nil}$)

Outline

Properties of Observation Congruence I

Corollary 8.4

For every $P, Q \in Prc$,

1. $P \sim Q \implies P \simeq Q$
2. $P \simeq Q \implies P \approx Q$

Proof.

1. since $\xrightarrow{\alpha} \subseteq \overset{\alpha}{\Longrightarrow}$ and $\sim \subseteq \approx$
2. since $\overset{\alpha}{\Longrightarrow} \subseteq \overset{\hat{\alpha}}{\Longrightarrow}$

□

Remark: this implies that

- processes with identical LTSs are $\simeq$-equivalent,
- $\simeq$-equivalent processes are (non-τ) trace equivalent, and
- $\simeq$ is (non-τ) deadlock sensitive.

Theorem 8.5

For every $P, Q \in Prc$,

$$P \simeq Q \iff P + R \approx Q + R \text{ for every } R \in Prc.$$

Proof.

on the board ☐

Remark: $\simeq$ is therefore the largest congruence contained in $\approx$

Properties of Observation Congruence III

Theorem 8.6

$\simeq$ *is an equivalence relation.*

Proof.

on the board $\square$

Theorem 8.7

$\simeq$ is a CCS congruence.

Proof.

omitted $\qquad\square$

Properties of Observation Congruence V

Theorem 8.8

For every $P, Q \in Prc$,

$$P \approx Q \iff P \simeq Q \text{ or } P \simeq \tau.Q \text{ or } \tau.P \simeq Q.$$

Proof.

see Exercise 5.3 □