Software Modeling and Verification
Lehrstuhl für Informatik 2
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Modeling Concurrent and Probabilistic Systems

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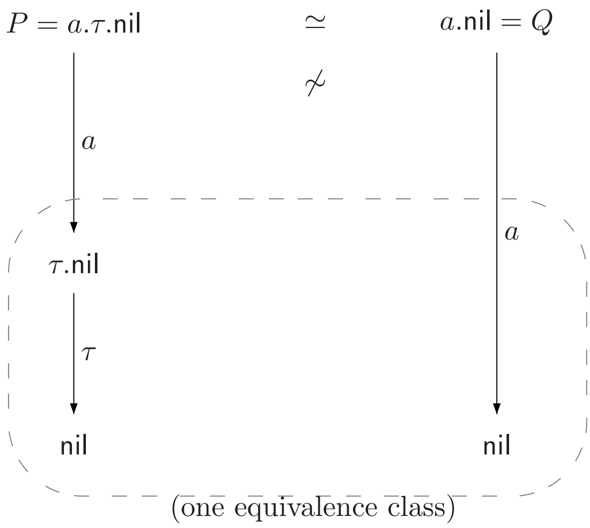
– Solution 5 –

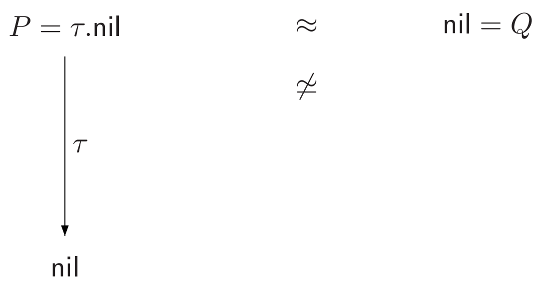
Exercise 4.4 **(4 points)**

$$\begin{array}{ccccc} P \sim Q & \Rightarrow & P \simeq Q & \Rightarrow & P \approx Q \\ & \not\Leftarrow & & \not\Leftarrow & \\ & \text{(a)} & & \text{(b)} & \end{array}$$

a) $P \simeq Q$ and $P \not\sim Q$:

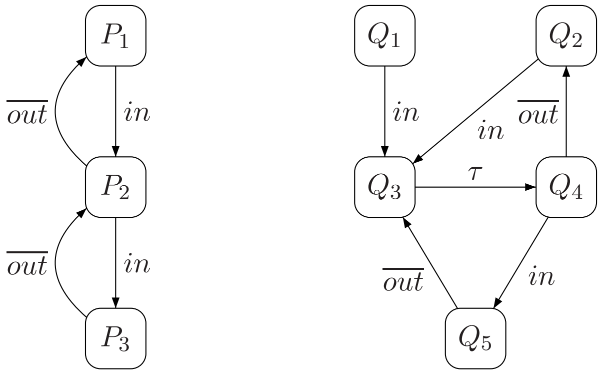


b) $P \approx Q$ and $P \not\simeq Q$:



Exercise 1 **(4 points)**

Specification: Implementation:



Partitioning algorithm:

(1) Initial partition $\pi = \{S\} = \{P_1, P_2, P_3, Q_1, \ldots, Q_5\}$

(2,3) Successor blocks:

$$\alpha^*(P) = \{B \in \pi \mid \exists P' \in B \text{ with } P \overset{\hat{\alpha}}{\Longrightarrow} P'\}$$

P	P_1	P_2	P_3	Q_1	Q_2	Q_3	Q_4	Q_5
$in^*(P)$	$\{S\}$	$\{S\}$	$\emptyset$	$\{S\}$	$\{S\}$	$\{S\}$	$\{S\}$	$\emptyset$
$\overline{out}^*(P)$	$\emptyset$	$\{S\}$	$\{S\}$	$\emptyset$	$\emptyset$	$\{S\}$	$\{S\}$	$\{S\}$
$\tau^*(P)$	$\{S\}$	$\{S\}$	$\{S\}$	$\{S\}$	$\{S\}$	$\{S\}$	$\{S\}$	$\{S\}$

(4,5) Decomposition: $\pi = \{\underbrace{\{P_1, Q_1, Q_2\}}_{B_1}, \underbrace{\{P_2, Q_3, Q_4\}}_{B_2}, \underbrace{\{P_3, Q_5\}}_{B_3}\}$

(2, 3) Successor blocks:

P	P_1	Q_1	Q_2	P_2	Q_3	Q_4	P_3	Q_5
$in^*(P)$	$\{B_2\}$	$\{B_2\}$	$\{B_2\}$	$\{B_3\}$	$\{B_3\}$	$\{B_3\}$	$\emptyset$	$\emptyset$
$\overline{out}^*(P)$	$\emptyset$	$\emptyset$	$\emptyset$	$\{B_1\}$	$\{B_1\}$	$\{B_1\}$	$\{B_2\}$	$\{B_2\}$
$\tau^*(P)$	$\{B_1\}$	$\{B_1\}$	$\{B_1\}$	$\{B_2\}$	$\{B_2\}$	$\{B_2\}$	$\{B_3\}$	$\{B_3\}$

(4,5) no change $\Rightarrow \hat{\pi} = \{B_1, B_2, B_3\}$

By correctness Theorem 9.1: $P_1, Q_1 \in B_1 \Rightarrow P_1 \approx Q_1$

Observational congruence:
Theorem 9.2: $P_1 \simeq Q_1 \Leftrightarrow \alpha^+(P_1) = \alpha^+(Q_1) \forall \alpha \in Act$
where $\alpha^+(P) = \{C \in \hat{\Pi} \mid \exists P' \in C \text{ with } P \overset{\alpha}{\Longrightarrow} P'\}$

Computation of the α^+-successor blocks:

P	P_1	Q_1
$in^+(P)$	$\{B_2\}$	$\{B_2\}$
$out^+(P)$	$\emptyset$	$\emptyset$
$\tau^+(P)$	$\emptyset$	$\emptyset$

Hence $P_1 \simeq Q_1$.

Exercise 2 (6 points)

We have to show:

$$P \approx Q \quad \Longleftrightarrow \quad P \simeq Q \text{ or } P \simeq \tau.Q \text{ or } \tau.P \simeq Q.$$

Proof:

$\Longleftarrow$ If $P \simeq Q$, then $P \approx Q$ by Corollary 10.4.
If $P \simeq \tau.Q$, then $P \approx \tau.Q$ by Corollary 10.4, and hence $P \approx Q$ by Lemma 9.5.
If $\tau.P \simeq Q$: analogously

$\Longrightarrow$ Let $P \approx Q$. We distinguish three cases:

a) $P \xrightarrow{\tau} P' \approx Q$ for some $P' \in \mathit{Proc}$: here $P \simeq \tau.Q$ since
   - if $P \xrightarrow{\tau} P'$, then $P \approx Q$ implies that there ex. Q' such that $Q \stackrel{\varepsilon}{\Rightarrow} Q'$ and $P' \approx Q'$. Hence $\tau.Q \stackrel{\tau}{\Rightarrow} Q'$ with $P' \approx Q$ q.e.d.
   - if $P \xrightarrow{\alpha} P'$ with $\alpha \neq \tau$, then $P \approx Q$ implies that there ex. Q' such that $Q \stackrel{\alpha}{\Rightarrow} Q'$ and $P' \approx Q'$. Hence $\tau.Q \stackrel{\alpha}{\Rightarrow} Q'$ with $P' \approx Q'$ q.e.d.
   - if $\tau.Q \xrightarrow{\tau} Q$, then $P \xrightarrow{\tau} P' \approx Q$ by above assumption q.e.d.

b) $Q \xrightarrow{\tau} Q' \approx P$ for some $Q' \in \mathit{Proc}$: here $\tau.P \simeq Q$ follows analogously to the previous case.

c) otherwise: here $P \simeq Q$ since
   - if $P \xrightarrow{\tau} P'$, then $P \approx Q$ implies that there ex. Q' such that $Q \stackrel{\varepsilon}{\Rightarrow} Q'$ and $P' \approx Q'$. Since $P' \not\approx Q$ (otherwise case (a) would apply), also $Q' \not\approx Q$. Hence $Q \stackrel{\tau}{\Rightarrow} Q'$ q.e.d.
   - if $P \xrightarrow{\alpha} P'$ with $\alpha \neq \tau$, then $P \approx Q$ implies that there ex. Q' such that $Q \stackrel{\alpha}{\Rightarrow} Q'$ and $P' \approx Q'$ q.e.d.
   - for $Q \xrightarrow{\tau} Q'$ and $Q \xrightarrow{\alpha} Q'$, similar arguments apply

Exercise 3 (4 points)

We know from Exercise 4.2: Turing machine $\mathcal{A} \mapsto$ process definition $P_{\mathcal{A}}$ such that the LTS of $P_{\mathcal{A}}$ represents the configurations of $\mathcal{A}$.

Concretely: State $q \in Q \mapsto$ process identifier $Control_q$.

Now: if $q \in F$ (final state), then extend $Control_q$ by $Control_q = \ldots + \overline{done}.\mathsf{nil}$.

Result: $\mathcal{A}$ halts in final state (undecidable)
$\Leftrightarrow P_{\mathcal{A}} \xrightarrow{\tau}{}^{*} \cdot \xrightarrow{\overline{done}} \mathsf{nil}$
$\Leftrightarrow P_{\mathcal{A}} \approx \overline{done}.\mathsf{nil}$
$\Rightarrow$ if we decided that weak bisimulation problem, we could decide the halting problem for TMs.