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Modeling Concurrent and Probabilistic Systems

Winter Term 07/08

– Solution 8 –

Exercise 1 (1 + 3 points)

a) Using structural congruence the proof proceeds as follows:

$$\begin{aligned}
P &= \mathsf{new}\ z\ \big((\overline{x}\,\langle y\rangle + z(w).\overline{w}\,\langle y\rangle) \mid\mid x(u).\overline{u}\,\langle v\rangle\ \mid\mid \overline{x}\,\langle z\rangle\big)\\
&\equiv \mathsf{new}\ z\ \big(x(u).\overline{u}\,\langle v\rangle\ \mid\mid (\overline{x}\,\langle y\rangle + z(w).\overline{w}\,\langle y\rangle) \mid\mid \overline{x}\,\langle z\rangle\big)\\
&\equiv x(u).\overline{u}\,\langle v\rangle\ \mid\mid \mathsf{new}\ z\ \big((\overline{x}\,\langle y\rangle + z(w).\overline{w}\,\langle y\rangle) \mid\mid \overline{x}\,\langle z\rangle\big)
\end{aligned}$$

b) P has three successors which can be derived by the reaction rules as follows:

- $P_1 = \mathsf{new}\ z\ \big(\mathsf{nil} \mid\mid \overline{y}\,\langle v\rangle\ \mid\mid \overline{x}\,\langle z\rangle\big)$:

$$\dfrac{\dfrac{\dfrac{}{(\overline{x}\langle y\rangle + z(w).\overline{w}\langle y\rangle) \mid\mid x(u).\overline{u}\langle v\rangle \to \mathsf{nil} \mid\mid \overline{y}\langle v\rangle}\,(\text{React})}{(\overline{x}\langle y\rangle + z(w).\overline{w}\langle y\rangle) \mid\mid x(u).\overline{u}\langle v\rangle \mid\mid \overline{x}\langle z\rangle \to \mathsf{nil} \mid\mid \overline{y}\langle v\rangle \mid\mid \overline{x}\langle z\rangle}\,(\text{Par})}{\mathsf{new}\,z((\overline{x}\langle y\rangle + z(w).\overline{w}\langle y\rangle) \mid\mid x(u).\overline{u}\langle v\rangle \mid\mid \overline{x}\langle z\rangle) \to \mathsf{new}\,z(\mathsf{nil} \mid\mid \overline{y}\langle v\rangle \mid\mid \overline{x}\langle z\rangle)}\,(\text{Res})$$

- $P_2 = \mathsf{new}\ z\ \big((\overline{x}\,\langle y\rangle + z(w).\overline{w}\,\langle y\rangle) \mid\mid \overline{z}\,\langle v\rangle\ \mid\mid \mathsf{nil}\big)$:

$$\dfrac{\dfrac{\dfrac{}{x(u).\overline{u}\langle v\rangle \mid\mid \overline{x}\langle z\rangle \to \overline{z}\langle v\rangle \mid\mid \mathsf{nil}}\,(\text{React})}{(\overline{x}\langle y\rangle + z(w).\overline{w}\langle y\rangle) \mid\mid x(u).\overline{u}\langle v\rangle \mid\mid \overline{x}\langle z\rangle \to (\overline{x}\langle y\rangle + z(w).\overline{w}\langle y\rangle) \mid\mid \overline{z}\langle v\rangle \mid\mid \mathsf{nil}}\,(\text{Par})}{\mathsf{new}\,z((\overline{x}\langle y\rangle + z(w).\overline{w}\langle y\rangle) \mid\mid x(u).\overline{u}\langle v\rangle \mid\mid \overline{x}\langle z\rangle) \to \mathsf{new}\,z((\overline{x}\langle y\rangle + z(w).\overline{w}\langle y\rangle) \mid\mid \overline{z}\langle v\rangle \mid\mid \mathsf{nil})}\,(\text{Res})$$

- $P_3 = \mathsf{new}\ z\ \big(\overline{v}\,\langle y\rangle\ \mid\mid \mathsf{nil} \mid\mid \mathsf{nil}\big)$:

$$\dfrac{\dfrac{\dfrac{}{(\overline{x}\langle y\rangle + z(w).\overline{w}\langle y\rangle) \mid\mid \overline{z}\langle v\rangle \to \overline{v}\langle y\rangle \mid\mid \mathsf{nil}}\,(\text{React})}{(\overline{x}\langle y\rangle + z(w).\overline{w}\langle y\rangle) \mid\mid \overline{z}\langle v\rangle \mid\mid \mathsf{nil} \to \overline{v}\langle y\rangle \mid\mid \mathsf{nil} \mid\mid \mathsf{nil}}\,(\text{Par})}{\mathsf{new}\,z((\overline{x}\langle y\rangle + z(w).\overline{w}\langle y\rangle) \mid\mid \overline{z}\langle v\rangle \mid\mid \mathsf{nil}) \to \mathsf{new}\,z(\overline{v}\langle y\rangle \mid\mid \mathsf{nil} \mid\mid \mathsf{nil})}\,(\text{Res})$$

Exercise 2 (4 points)

Let Q be a process and $x \notin \mathsf{fn}(Q)$. Then $\mathsf{new}\ x\ Q \equiv Q$.
Proof:

$$\begin{aligned}
Q &\equiv Q \mid\mid \mathsf{nil} && \text{structural congruence (3)}\\
&\equiv Q \mid\mid \mathsf{new}\ x\ \mathsf{nil} && \text{structural congruence (4)}\\
&\equiv \mathsf{new}\ x\ (Q \mid\mid \mathsf{nil}) && \text{structural congruence (4)}\\
&\equiv \mathsf{new}\ x\ Q && \text{as } P \mid\mid \mathsf{nil} \equiv P
\end{aligned}$$

By transitive closure of $\equiv$, the claim follows.

Exercise 3 **(2 points)**

$$
\begin{aligned}
&x(z).\overline{y}\langle z\rangle \mid\mid !(\mathsf{new}\, y(\overline{x}\langle y\rangle.Q))\\
\equiv &x(z).\overline{y}\langle z\rangle \mid\mid \mathsf{new}\, y(\overline{x}\langle y\rangle.Q) \mid\mid !(\mathsf{new}\, y(\overline{x}\langle y\rangle.Q))\\
\equiv &x(z).\overline{y}\langle z\rangle \mid\mid \mathsf{new}\, \tilde{y}(\overline{x}\langle \tilde{y}\rangle.Q) \mid\mid !(\mathsf{new}\, y(\overline{x}\langle y\rangle.Q))\\
\equiv &\mathsf{new}\, \tilde{y}(x(z).\overline{y}\langle z\rangle \mid\mid \overline{x}\langle \tilde{y}\rangle.Q) \mid\mid !(\mathsf{new}\, y(\overline{x}\langle y\rangle.Q))\\
\equiv &\mathsf{new}\, \tilde{y}(x(z).\overline{y}\langle z\rangle \mid\mid \overline{x}\langle \tilde{y}\rangle.Q \mid\mid !(\mathsf{new}\, y(\overline{x}\langle y\rangle.Q)))
\end{aligned}
$$

$$
\text{(Res)}\dfrac{\text{(Par)}\dfrac{\text{(React)}\dfrac{}{x(z).\overline{y}\langle z\rangle \mid\mid \overline{x}\langle \tilde{y}\rangle.Q \to \overline{y}\langle \tilde{y}\rangle \mid\mid Q}}{x(z).\overline{y}\langle z\rangle \mid\mid \overline{x}\langle \tilde{y}\rangle.Q \mid\mid !(\mathsf{new}\, y(\overline{x}\langle y\rangle.Q)) \to \overline{y}\langle \tilde{y}\rangle \mid\mid Q \mid\mid !(\mathsf{new}\, y(\overline{x}\langle y\rangle.Q))}}{\mathsf{new}\, \tilde{y}(x(z).\overline{y}\langle z\rangle \mid\mid \overline{x}\langle \tilde{y}\rangle.Q \mid\mid !(\mathsf{new}\, y(\overline{x}\langle y\rangle.Q))) \to \mathsf{new}\, \tilde{y}(\overline{y}\langle \tilde{y}\rangle \mid\mid Q \mid\mid !(\mathsf{new}\, y(\overline{x}\langle y\rangle.Q)))}
$$

Exercise 4 **(4 points)**

$$
x(y_1y_2).P \mid\mid \overline{x}\langle z_1z_2\rangle.Q \mid\mid \overline{x}\langle z_1'z_2'\rangle.Q'
$$

$$
\rightsquigarrow
$$

$$
\underbrace{x(w).w(y_1).w(y_2).P}_{=:P_1} \mid\mid \underbrace{\mathsf{new}\, w(\overline{x}\langle w\rangle.\overline{w}\langle z_1\rangle.\overline{w}\langle z_2\rangle.Q}_{=:P_2} \mid\mid \underbrace{\mathsf{new}\, w'(\overline{x}\langle w'\rangle.\overline{w'}\langle z_1'\rangle.\overline{w'}\langle z_2'\rangle.Q'}_{=:P_3}
$$

We will do some reductions as follows: Since $P_1 \mid\mid P_2 \equiv \mathsf{new}\, w(P_1 \mid\mid \overline{x}\langle w\rangle.\overline{w}\langle z_1\rangle.\overline{w}\langle z_2\rangle.Q)$,

$$
\begin{aligned}
P_1 \mid\mid P_2 \mid\mid P_3 \quad &\longrightarrow \quad \mathsf{new}\, w(w(y_1).w(y_2).P \mid\mid \overline{w}\langle z_1\rangle.\overline{w}\langle z_2\rangle.Q) \mid\mid P_3\\
&\longrightarrow \quad \mathsf{new}\, w(w(y_2).P[y_1/z_1] \mid\mid \overline{w}\langle z_2\rangle.Q) \mid\mid P_3\\
&\overset{\mathsf{new}\, w\, Q \equiv Q}{\longrightarrow} \quad P[y_1/z_1, y_2/z_2] \mid\mid Q \mid\mid P_3
\end{aligned}
$$

and furthermore

$$
\begin{aligned}
P_1 \mid\mid P_2 \mid\mid P_3 \quad &\longrightarrow \quad \mathsf{new}\, w'(w'(y_1).w'(y_2).P \mid\mid \overline{w'}\langle z_1'\rangle.\overline{w'}\langle z_2'\rangle.Q') \mid\mid P_2\\
&\longrightarrow \quad \mathsf{new}\, w'(w'(y_2).P[y_1/z_1'] \mid\mid \overline{w'}\langle z_2'\rangle.Q') \mid\mid P_2\\
&\overset{\mathsf{new}\, w'\, Q' \equiv Q'}{\longrightarrow} \quad P[y_1/z_1', y_2/z_2'] \mid\mid Q' \mid\mid P_2
\end{aligned}
$$