

# Modeling Concurrent and Probabilistic Systems

Summer Term 09

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## – Series 3 –

Hand in until May 20 before the exercise class.

### Exercise 1 (3 + 1 points)

In analogy to the buffer, this exercise deals with the *semaphore* data structure which can be regarded as a buffer for identical entries.

a) An  $n$ -ary semaphore ( $n \geq 1$ ) is given by the following specification:

$$\begin{aligned} Sem_0^n(get, put) &= get.Sem_1^n(get, put) \\ Sem_k^n(get, put) &= get.Sem_{k+1}^n(get, put) + put.Sem_{k-1}^n(get, put) \quad (0 < k < n) \\ Sem_n^n(get, put) &= put.Sem_{n-1}^n(get, put). \end{aligned}$$

Prove that it can also be specified by composing  $n$  parallel instances of a unary semaphore:

$$\begin{aligned} S^n(get, put) &= \underbrace{S_0(get, put) \parallel S_0(get, put) \parallel \cdots \parallel S_0(get, put)}_{n \text{ times}} \\ S_0(get, put) &= get.S_1(get, put) \\ S_1(get, put) &= put.S_0(get, put). \end{aligned}$$

To this aim, show that  $Sem_0^n \sim S^n$  by constructing an appropriate strong bisimulation relation.

b) Show that the two variants of the two place buffer given in the lecture are not strongly bisimilar.

### Exercise 2 (2 points)

Prove or disprove the following strong bisimulation equivalences between process terms:

a)  $P + \text{nil} \sim P$   
 b)  $P \parallel (Q \parallel R) \sim (P \parallel Q) \parallel R$

### Exercise 3 (3 points)

Prove that it is undecidable whether a given CCS process definition induces a finite or an infinite transition system.

*Hint:* a turing tape can be simulated by means of two stacks.