Software Modeling and Verification
Lehrstuhl für Informatik 2
RWTH Aachen University

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Modeling Concurrent and Probabilistic Systems

Summer Term 09

– Series 6 –

Hand in until June 24 before the exercise class.

Exercise 1 ($0.5 \times 6 = 3$ points)

Let X be the trial time when the first *failure* occurs in an infinite sequence of Bernoulli experiments with probability p for success. Define a stochastic process for random variable X and give:

a) the state space of X;

b) the probability distribution of X;

c) show that the probability distribution of X sums up to 1;

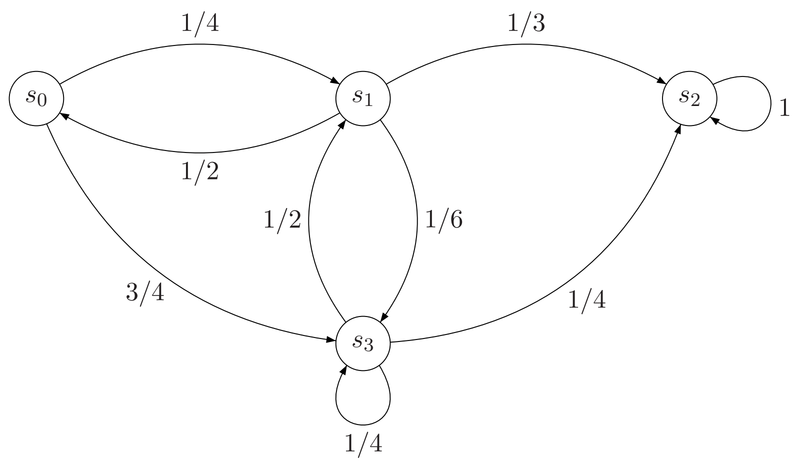
d) the distribution function of X;

e) the expected value of X;

f) the variance of X.

Exercise 2 ($1 \times 3 = 3$ points)

Given the DTMC $\mathcal{D}$ as follows:



a) Compute the probability of going from s_0 to s_3 in *exactly* 3 steps;
(*Hints: by the end of the* 3*rd step the system is in state* 3.)

b) Compute the probability of going from s_0 to s_3 in *at most* 3 steps;
(*Hints: by the end of the* 3*rd step the system has been in state* 3.)

c) Compute the probability of being in state s_2 after 3 steps when the initial distribution is uniform over all states.

Exercise 3 **($1 \times 3 = 3$ points)**

Consider the DTMC with set of states $\{s_1, \ldots, s_7\}$ given by the following transition probability matrix:

$$\begin{pmatrix} 0 & 1/3 & 2/3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/3 & 1/6 & 1/2 & 0 & 0 \\ 0 & 3/5 & 0 & 0 & 0 & 0 & 2/5 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3/4 & 0 & 0 & 1/4 \\ 0 & 0 & 0 & 1/2 & 0 & 0 & 1/2 \end{pmatrix}$$

Determine its recurrent, transient and periodic states.

Exercise 4 **($1 + 2 = 3$ points)**

a) Given a DTMC whose states are positive integers. From state i, the probability of going to state $i + 1$ is $i/(i + 1)$. With probability $1/(i + 1)$, the chain returns to state 1.

  (a) Is state 1 recurrent or not?

  (b) Is state 1 positive recurrent or not?

b) Suppose the above DTMC is cut to have the first 4 states, instead of having a transition to state 5 with probability 4/5, the fourth state has a self-loop with probability 4/5 and with probability 1/5 it goes back to state 1. The other 3 states remain the same.

  (a) Compute the limiting distribution of this finite DTMC;

  (b) Compute the stationary distribution for state 1 by using $m_s = \sum_{n=1}^{\infty} n \cdot f_s(n)$.