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Modeling Concurrent and Probabilistic Systems

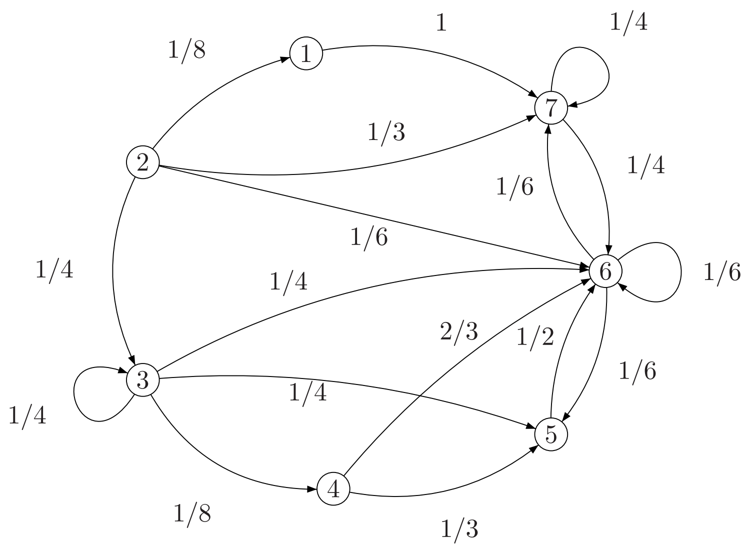
Summer Term 09

– Series 7 –

Hand in until July 1 before the exercise class.

Exercise 1 (3 points)

A *fully probabilistic system* (FPS) is a DTMC, but with a relaxed condition: $\sum_{s' \in S} \mathbf{P}(s, s') \in [0, 1]$ for all $s \in S$, where for DTMCs the condition is $\sum_{s' \in S} \mathbf{P}(s, s') = 1$ for all $s \in S$. The bisimulation relation on an FPS is defined in the same way as in a DTMC. Now consider the FPS $\mathcal{D}$ as follows:



a) Determine $\mathcal{D}/\sim_p$.

b) For each $C \in S/\sim_p$, compute $\underline{p'}_C(3)$, given the initial distribution $\underline{p}(0) = (1/5, 0, 2/15, 1/3, 1/6, 1/9, 1/18)$.

c) Compute the 3-step transient probability distribution in $\mathcal{D}/\sim_p$, given the same initial distribution as in b).

Exercise 2 (5 points)

Let $\mathcal{D} = (S, \mathbf{P})$ be a DTMC and $T \subseteq S, L \subseteq S$. For $s \in S$, let

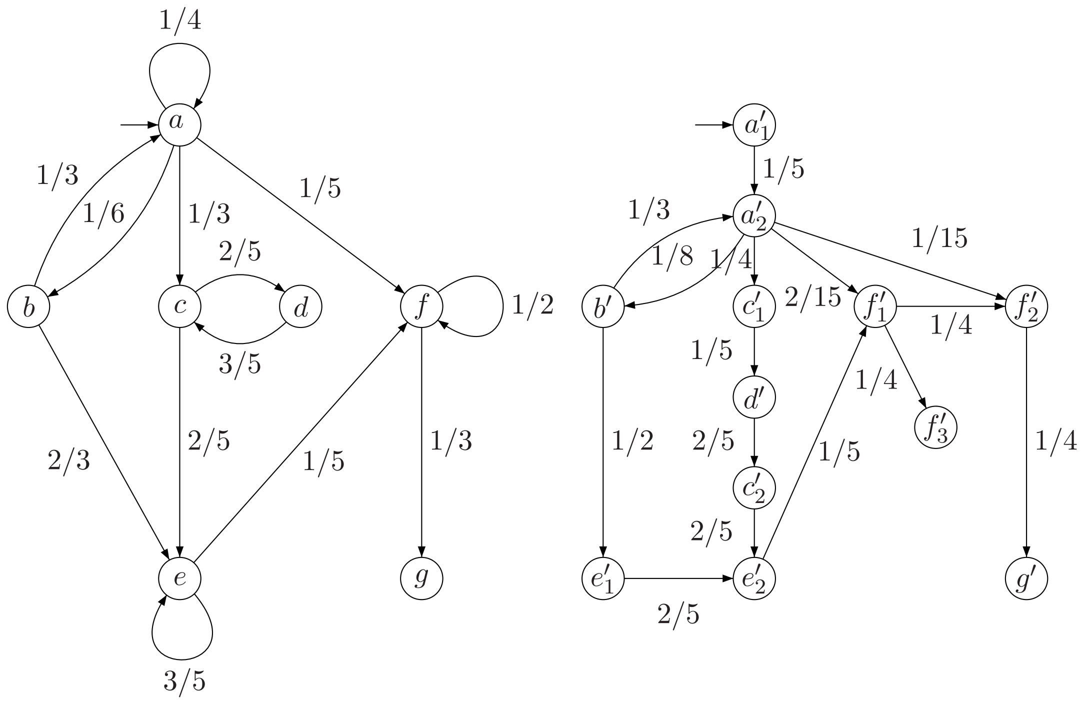
$$Prob(s, L, T) = \Pr\{X(i) \in T \text{ for some } i \geq 0 \text{ and } X(j) \in L \text{ for all } 0 \leq j < i \mid X(0) = s\}$$

a) Give a recurrent equation for $Prob(s, L, T)$.

b) Now let $T \in S/\sim_p$ and $L \in S/\sim_p$. Show that $s \sim_p s'$ implies $Prob(s, L, T) = Prob(s', L, T)$.

Exercise 3 **(2 points)**

Given two FPSs $\mathcal{D}_l$ and $\mathcal{D}_r$ as follows:



Do we have:

a) $(\mathcal{D}_l, a) \sqsubseteq_p (\mathcal{D}_r, a'_1)$?

b) $(\mathcal{D}_r, a'_1) \sqsubseteq_p (\mathcal{D}_l, a)$?