

Modeling Concurrent and Probabilistic Systems

Summer Term 09

— Series 8 —

Hand in until July 13 before the exercise class.

Exercise 1

(3 points)

Let $\mathcal{D} = (S, \mathbf{P})$ be an FPS and R an equivalence relation on S . R is a *weak bisimulation* on \mathcal{D} if for all $s_1 R s_2$:

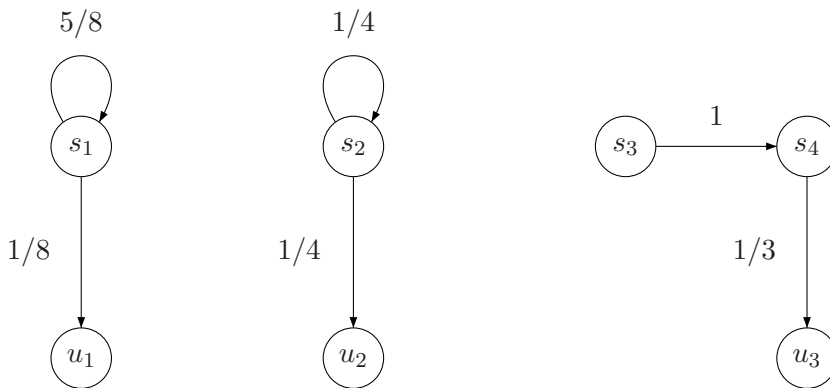
a) If $\mathbf{P}(s_i, [s_i]_R) < 1$ for $i = 1, 2$ then for all $C \in S/R$, $C \neq [s_1]_R = [s_2]_R$:

$$\frac{\mathbf{P}(s_1, C)}{1 - \mathbf{P}(s_1, [s_1]_R)} = \frac{\mathbf{P}(s_2, C)}{1 - \mathbf{P}(s_2, [s_2]_R)}.$$

b) s_1 can reach a state outside $[s_1]_R$ iff s_2 can reach a state outside $[s_2]_R$.

s_1 and s_2 in \mathcal{D} are weakly bisimilar, denoted $s_1 \approx s_2$, iff there exists a weak bisimulation R on \mathcal{D} such that $s_1 R s_2$.

Consider the FPS depicted in the figure. Find an equivalence relation $R \subseteq S \times S$ which is a weak bisimulation. Prove your answer! (Note: the depicted FPS is a single FPS!)



Exercise 2

(4 points)

Determine the semantics of X in terms of a probabilistic transition system:

$$X = \alpha.X \oplus_{\frac{1}{4}} (\beta.Y \oplus_{\frac{2}{3}} Y)$$

$$Y = \alpha.Y \oplus_{\frac{1}{3}} (\beta.X \oplus_{\frac{3}{4}} X)$$

Exercise 3

(3 points)

For each of the following specifications, determine the semantics of X in terms of a probabilistic transition system with indices:

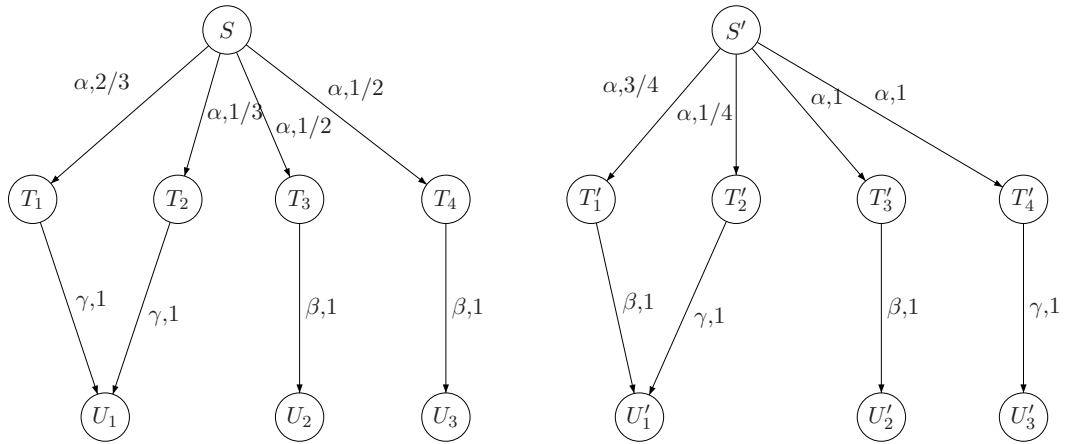
a) $X = \alpha.([1/6]\alpha.\mathbf{0} + [1/3]\beta.\mathbf{0} + [1/2]\gamma.\mathbf{0}) \parallel \{\gamma\} \oplus_{3/4} \alpha.\mathbf{0}$

b) $X = \alpha.\mathbf{0} \oplus_{1/3} (\alpha.\alpha.\mathbf{0} \otimes X)$

Exercise 4

(2 points)

Consider two process algebras PA_1, PA_2 given by the following figure:



Do we have

- $S \sim_p S'$?
- $S \sim_{cp} S'$?