

# Modeling Concurrent and Probabilistic Systems

Summer Term 09

## – Series 8 –

Hand in until July 13 before the exercise class.

### Exercise 1

(3 points)

Let  $\mathcal{D} = (S, \mathbf{P})$  be an FPS and  $R$  an equivalence relation on  $S$ .  $R$  is a *weak bisimulation* on  $\mathcal{D}$  if for all  $s_1 R s_2$ :

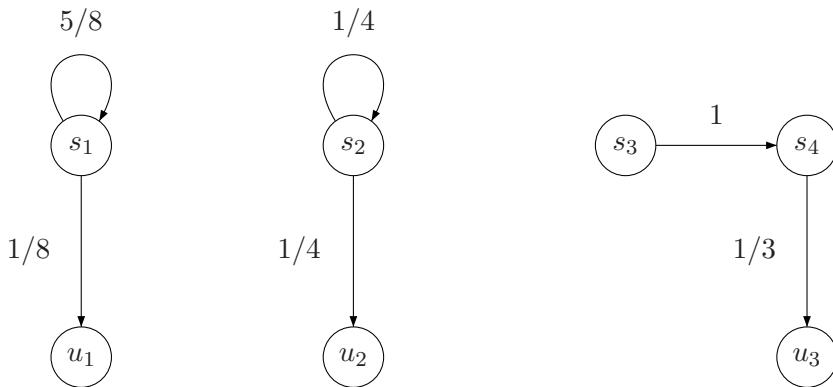
- a) If  $\mathbf{P}(s_i, [s_i]_R) < 1$  for  $i = 1, 2$  then for all  $C \in S/R$ ,  $C \neq [s_1]_R = [s_2]_R$ :

$$\frac{\mathbf{P}(s_1, C)}{1 - \mathbf{P}(s_1, [s_1]_R)} = \frac{\mathbf{P}(s_2, C)}{1 - \mathbf{P}(s_2, [s_2]_R)}.$$

- b)  $s_1$  can reach a state outside  $[s_1]_R$  iff  $s_2$  can reach a state outside  $[s_2]_R$ .

$s_1$  and  $s_2$  in  $\mathcal{D}$  are weakly bisimilar, denoted  $s_1 \approx s_2$ , iff there exists a weak bisimulation  $R$  on  $\mathcal{D}$  such that  $s_1 R s_2$ .

Consider the FPS depicted in the figure. Find an equivalence relation  $R \subseteq S \times S$  which is a weak bisimulation. Prove your answer! (Note: the depicted FPS is a single FPS!)



### Exercise 2

(4 points)

Determine the semantics of  $X$  in terms of a probabilistic transition system:

$$X = \alpha.X \oplus_{\frac{1}{4}} (\beta.Y \oplus_{\frac{2}{3}} Y)$$

$$Y = \alpha.Y \oplus_{\frac{1}{3}} (\beta.X \oplus_{\frac{3}{4}} X)$$

### Exercise 3

(3 points)

For each of the following specifications, determine the semantics of  $X$  in terms of a probabilistic transition system with indices:

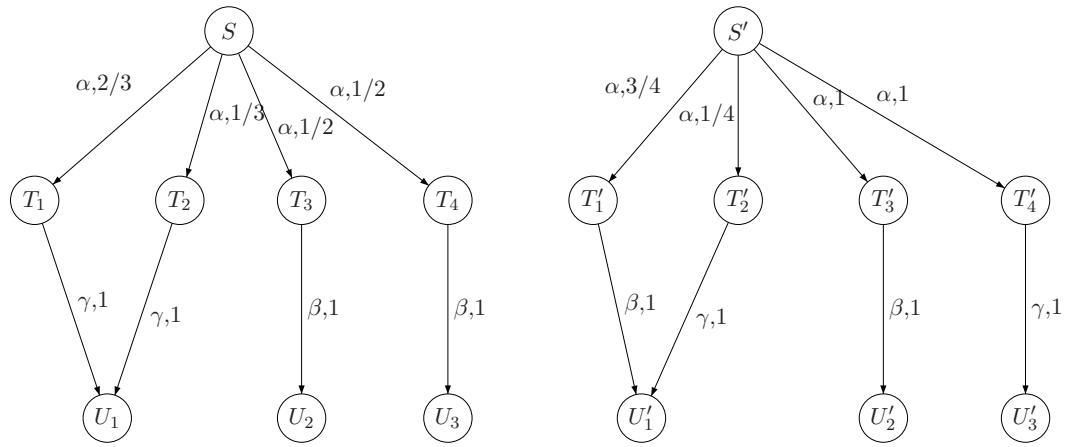
a)  $X = \alpha.(([1/6]\alpha.\mathbf{0} + [1/3]\beta.\mathbf{0} + [1/2]\gamma.\mathbf{0}) \setminus\setminus \{\gamma\}) \oplus_{3/4} \alpha.\mathbf{0}$

b)  $X = \alpha.\mathbf{0} \oplus_{1/3} (\alpha.\alpha.\mathbf{0} \otimes X)$

### Exercise 4

(2 points)

Consider two process algebras  $PA_1, PA_2$  given by the following figure:



Do we have

- $S \sim_p S'?$
- $S \sim_{cp} S'?$