

Modeling Concurrent and Probabilistic Systems

Summer Term 09

– Series 9 –

Hand in until July 15 before the exercise class.

Exercise 1

(2 points)

Let X and Y be two probabilistic processes given by:

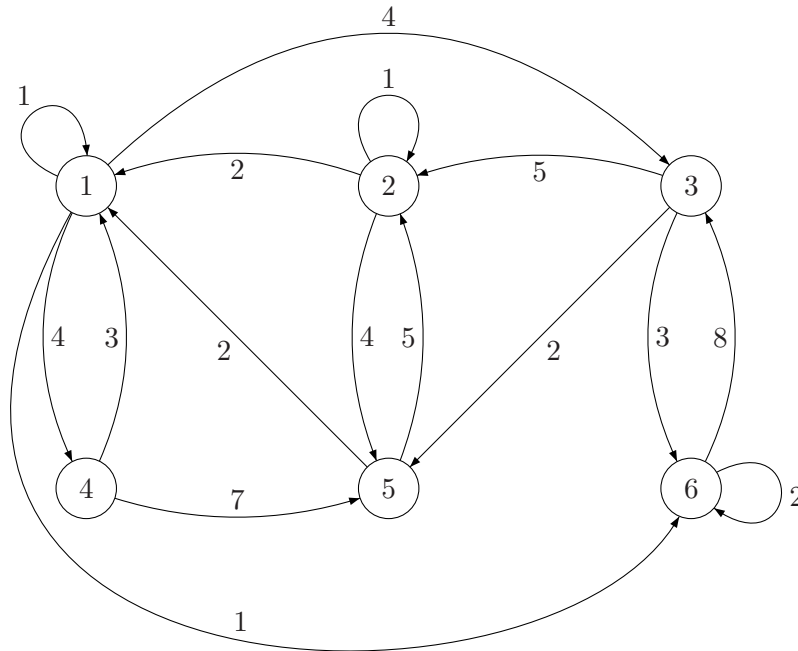
$$\begin{aligned} X &= \alpha.(X \oplus_{1/3} X') \\ X' &= \beta.(X' \oplus_{1/7} \mathbf{0}) \\ Y &= \beta.(Y \oplus_{1/5} \gamma.Y) + \alpha.(Y \oplus_{1/4} \zeta.Y) \end{aligned}$$

Compute the asynchronous parallel composition $X ||_{\{\alpha, \beta\}} Y$.

Exercise 2

(4 points)

Consider the CTMC \mathcal{C} , which is given by:



and its starting distribution $\underline{p}(0) = (0.1, 0.2, 0, 0, 0.4, 0.3)$.

1. Determine \mathcal{C} / \sim_m .
2. Determine the steady-state distribution of \mathcal{C} .

Exercise 3**(4 points)**

A professor supervises three Ph.D. students who all need quite a bit of advice. When any of these students visits the professor, the time to the next visit has an exponential distribution with a mean of 8 hours. The time for the professor to advise the students has a mean value of $1/2$ hour.

All times in this problem have exponential distributions. Students visit the professor one at a time. If the professor is busy, the students wait outside his office. And the students are treated in a first-come-first-served manner.

1. Construct the CTMC that describes the situation.
2. At steady state what proportion of the professor's time does he have to himself (without students)?
3. Give the expression of computing the transient distribution of the CTMC at time \hat{t} . Suppose initially the professor is free. Use the uniformised CTMC.
4. If now comes a new Ph.D. student, who has the same arrival rate as the others, but requires a mean time of one hour with the professor. Do 1 again.

Exercise 4**(4 points)**

To prove: For CTMC \mathcal{C} with $s_1, s_2 \in S$: $s_1 \approx_m s_2$ implies $s_1 \approx_p s_2$ in \mathcal{C} 's embedded DTMC $emb(\mathcal{C})$.

(Note that the definition of weak probabilistic bisimulation \approx_p is given in Ex. 08.)