Modeling Concurrent and Probabilistic Systems

Lecture 10: The Alternating-Bit Protocol

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Outline

The Setting

Goal: design of a communication protocol which guarantees reliable data transfer over unreliable channels

Overview of system "architecture":

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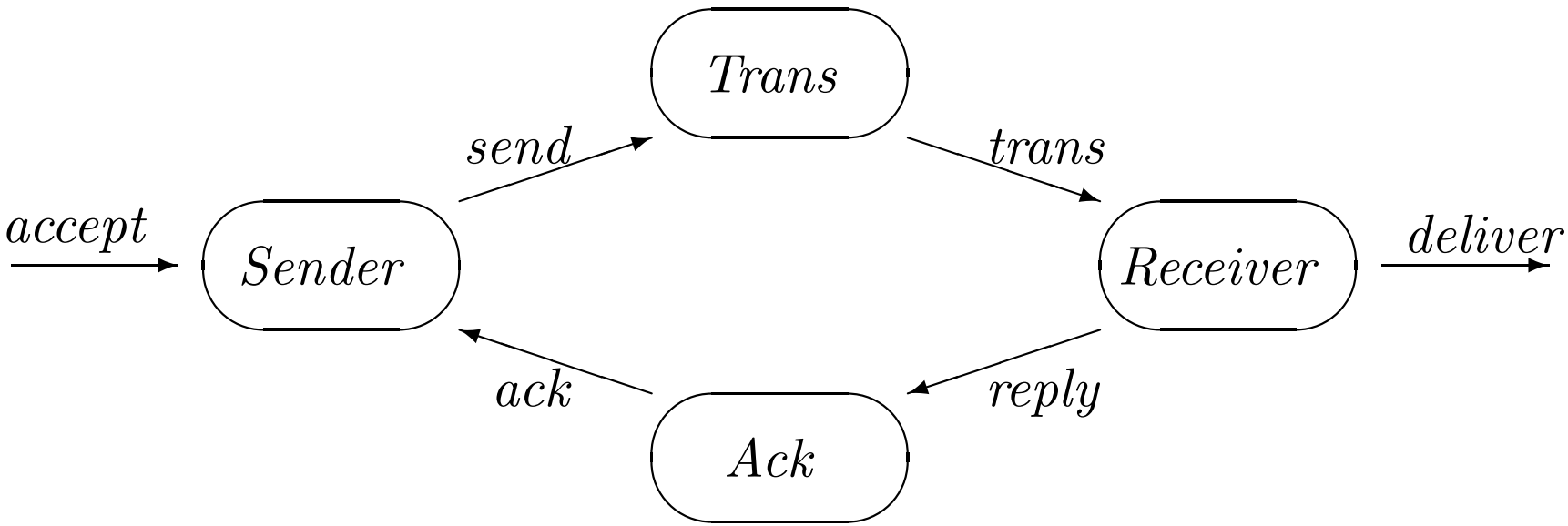
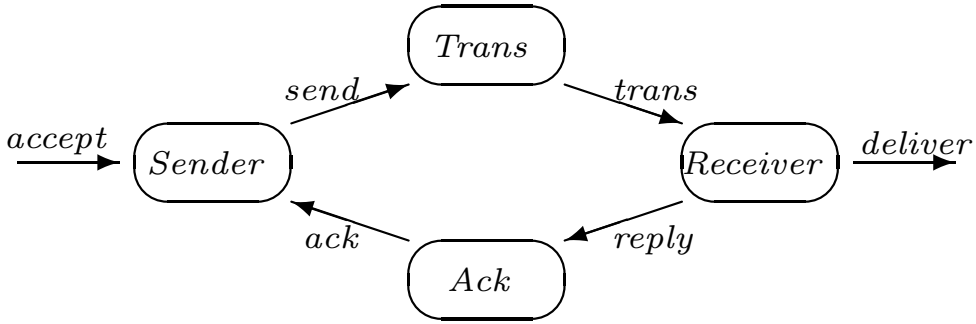


Working Principle



- *Sender* transfers data (from a given finite set D) to *Receiver* using channel *Trans*
- *Receiver* confirms reception via *Ack*
- Properties of channels:
  - unidirectional data transfer
  - capacity: one message
    ($\Longrightarrow$ sequential, i.e., respects order of messages)
  - detection of transmission errors
    (loss/duplication/corruption of messages)
  - errors reported to target station

Idea: use redundancy (additional control bit) to ensure safeness of data transfer

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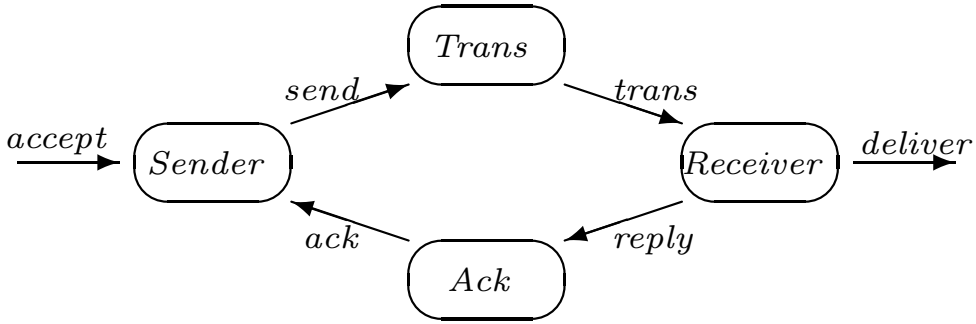

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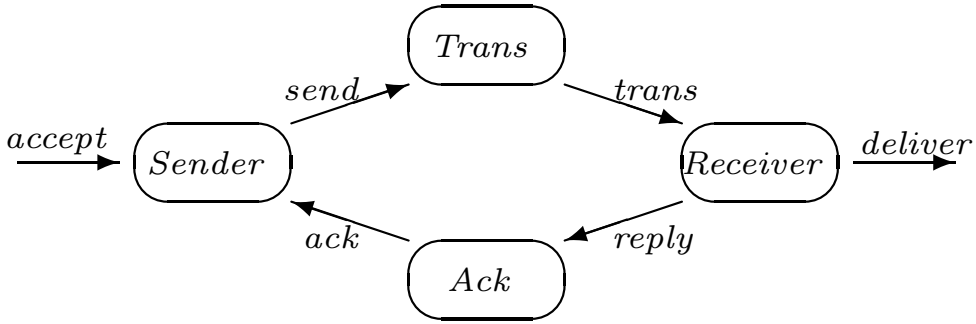

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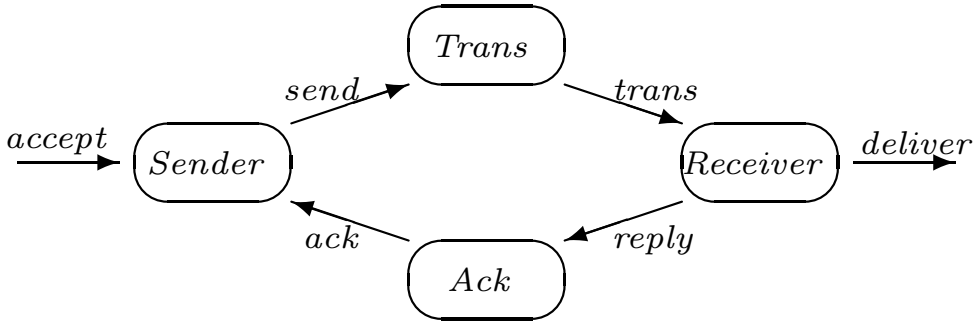

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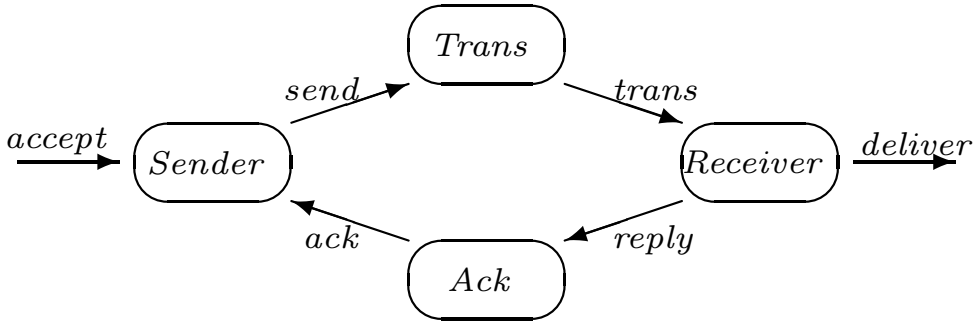
Modeling of Channels



- *Trans* transmits frames of the following form:

$$F := \{db \mid d \in D, b \in \{0,1\}\} \qquad \text{(finite)}$$

It detects transmission errors and reports it to *Receiver*:

$$Trans = \sum_{f \in F} send_f.(\underbrace{\overline{trans_f}.Trans}_{\text{successful}} + \underbrace{\overline{trans_\perp}.Trans}_{\text{error}})$$

- *Ack* behaves like *Trans* but transmits only control bits:

$$Ack = \sum_{b \in \{0,1\}} reply_b.(\underbrace{\overline{ack_b}.Ack}_{\text{successful}} + \underbrace{\overline{ack_\perp}.Ack}_{\text{error}})$$

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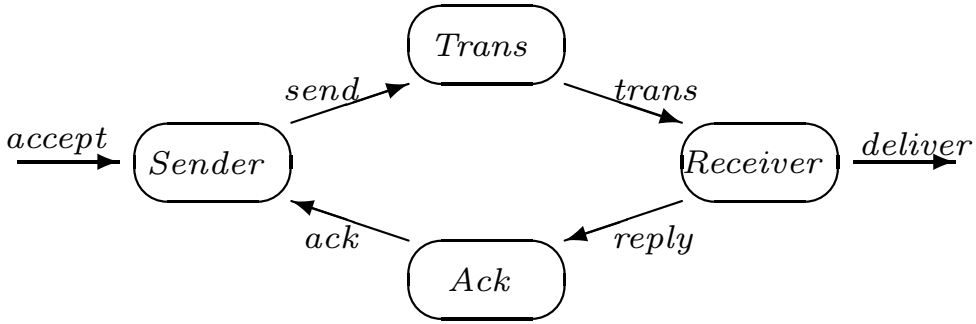

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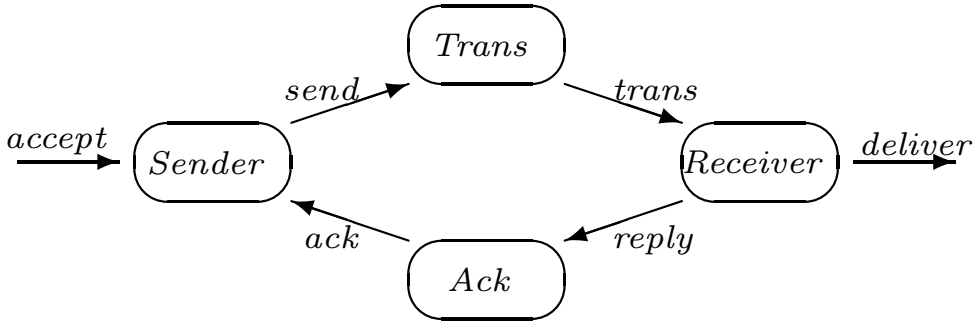
Design Goal



Under the above side conditions, give CCS implementations of *Sender* and *Receiver* such that the overall system works correctly, i.e., behaves like a one-element buffer:

$$Buffer(\overrightarrow{accept}, \overrightarrow{deliver}) = \sum_{d \in D} accept_d.Buffer_d(\overrightarrow{accept}, \overrightarrow{deliver})$$

$$Buffer_d(\overrightarrow{accept}, \overrightarrow{deliver}) = \overline{deliver_d}.Buffer(\overrightarrow{accept}, \overrightarrow{deliver})$$

where

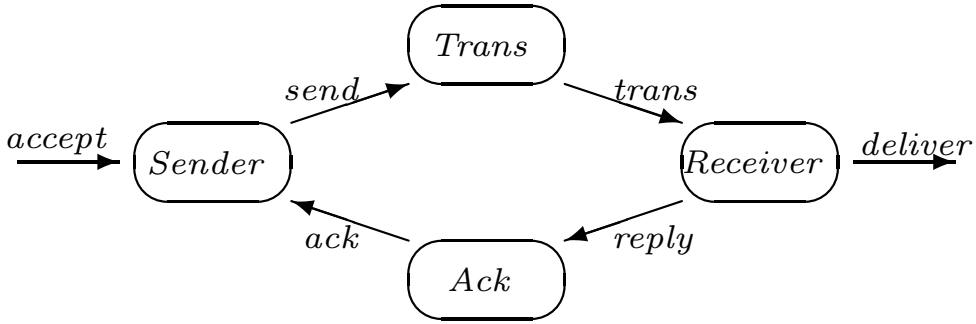
$$\overrightarrow{accept} := (accept_{d_1}, \ldots, accept_{d_n})$$

$$\text{and } \overrightarrow{deliver} := (deliver_{d_1}, \ldots, deliver_{d_n})$$

$$\text{for } D = \{d_1, \ldots, d_n\}$$

Outline

Implementation of Sender



Sender accepts $d \in D$ via $accept_d$ and repeatedly sends frames of the form $d0$ over *Trans* until it receives the acknowledgment 0 over *Ack*. For the next data item, control bit 1 is used and so on ($\Longrightarrow$ "Alternating-Bit Protocol").

Formally, for $b \in \{0,1\}$ and $d \in D$:

$$Sender = Sender_0$$

$$Sender_b = \sum_{d \in D} accept_d.Send_{db}$$

$$Send_{db} = \overline{send_{db}}.Wait_{db}$$

$$Wait_{db} = \underbrace{ack_b.Sender_{1-b}}_{\text{successful}} + \underbrace{ack_{1-b}.Send_{db} + nak.Send_{db}}_{\text{error}}$$

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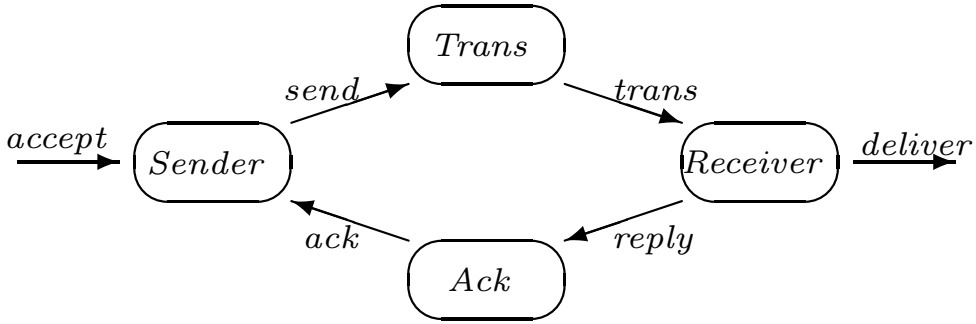

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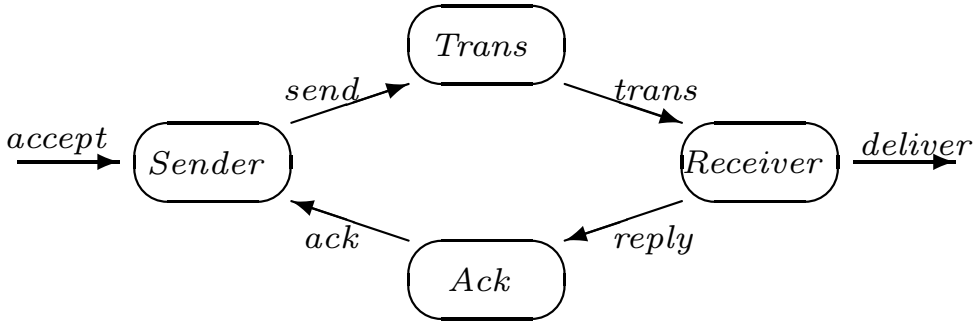

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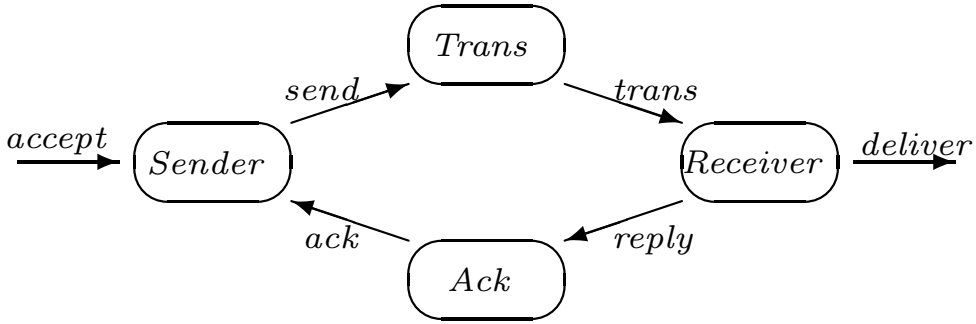

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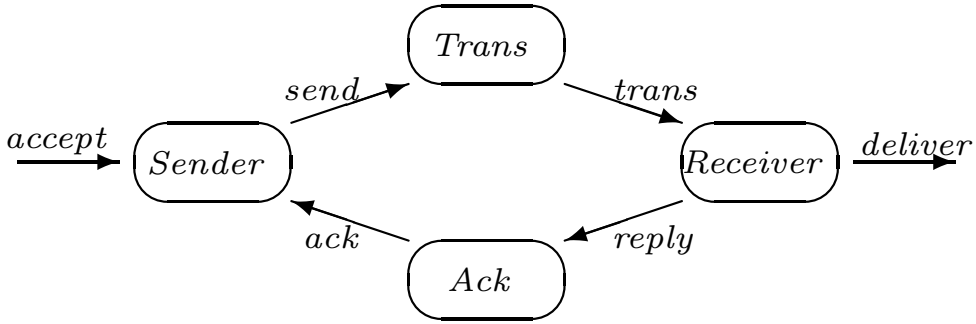

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Implementation of Receiver

Receiver gets frames of the form db or $\perp$. In the first case, if b has the expected value, d is forwarded via $deliver_d$, and b is returned via Ack. Otherwise the transmission is re-initiated by returning the "wrong" control bit $1 - b$ to *Sender*.

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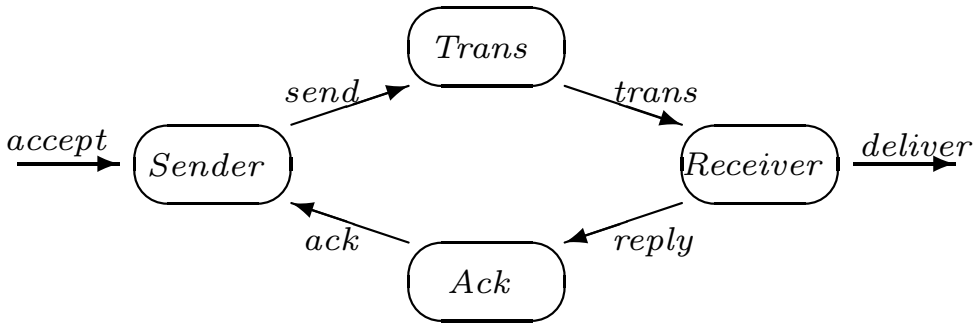
The Overall System



The overall system is given by

$$ABP(\overrightarrow{accept}, \overrightarrow{deliver}) = \mathsf{new}\, L\, (Sender \parallel Trans \parallel Ack \parallel Receiver)$$

where

$$L := \{send_{db}, trans_{db}, reply_b, ack_b \mid db \in F\} \cup \{trans_\perp, ack_\perp\}$$

Outline

Correctness of ABP I

Theorem 10.1

$ABP(\overrightarrow{accept}, \overrightarrow{deliver}) \simeq Buffer(\overrightarrow{accept}, \overrightarrow{deliver})$

Remark: because of internal τ-steps in ABP, $ABP \sim Buffer$ cannot hold.

Proof.

1. Construct transition system of $ABP(\overrightarrow{accept}, \overrightarrow{deliver})$ (next slide; $S = Sender$, $W = Wait$, $T = Trans$, $A = Ack$, $R = Receiver/Reply$, $d, e \in D$; without restrictions)
2. Show that $ABP(\overrightarrow{accept}, \overrightarrow{deliver}) \approx Buffer(\overrightarrow{accept}, \overrightarrow{deliver})$
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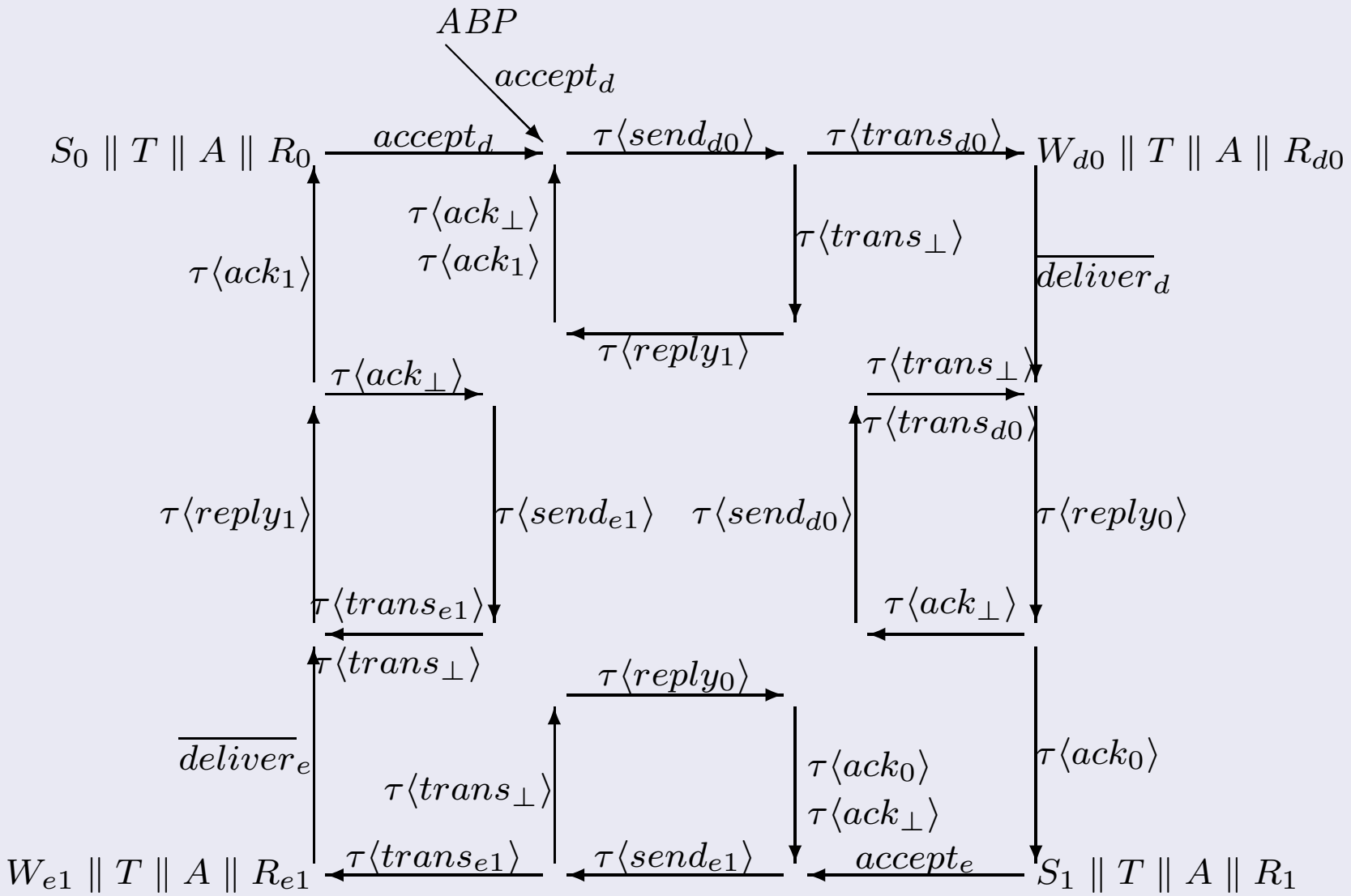

Correctness of ABP II

Proof (continued).