Modeling Concurrent and Probabilistic Systems

Lecture 11: Extensions of the Alternating-Bit Protocol

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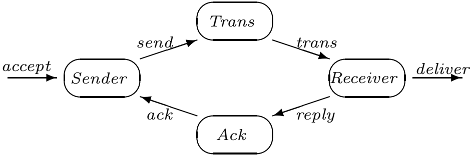
Outline

Repetition: The Alternating-Bit Protocol

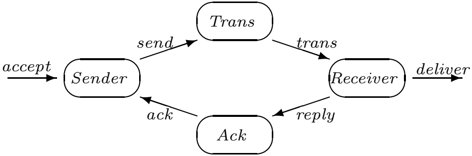


The overall system is given by

$$ABP(\overrightarrow{accept}, \overrightarrow{deliver}) = \mathsf{new}\, L\, (Sender \parallel Trans \parallel Ack \parallel Receiver)$$

where

$$L := \{send_{db}, trans_{db}, reply_b, ack_b \mid db \in F\} \cup \{trans_\perp, ack_\perp\}$$



- *Trans* transmits frames of the following form:

$$F := \{db \mid d \in D, b \in \{0,1\}\} \qquad \text{(finite)}$$

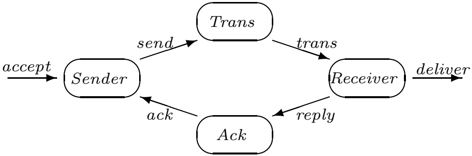
It detects transmission errors and reports it to *Receiver*:

$$Trans = \sum_{f \in F} send_f.(\underbrace{\overline{trans_f}.Trans}_{\text{successful}} + \underbrace{\overline{trans_\perp}.Trans}_{\text{error}})$$

- *Ack* behaves like *Trans* but transmits only control bits:

$$Ack = \sum_{b \in \{0,1\}} reply_b.(\underbrace{\overline{ack_b}.Ack}_{\text{successful}} + \underbrace{\overline{ack_\perp}.Ack}_{\text{error}})$$

Repetition: Implementation of Sender



Sender accepts $d \in D$ via $accept_d$ and repeatedly sends frames of the form $d0$ over *Trans* until it receives the acknowledgment 0 over *Ack*. For the next data item, control bit 1 is used and so on ($\implies$ "Alternating-Bit Protocol").

Formally, for $b \in \{0,1\}$ and $d \in D$:

$$
\begin{aligned}
Sender &= Sender_0 \\
Sender_b &= \sum_{d \in D} accept_d . Send_{db} \\
Send_{db} &= \overline{send_{db}} . Wait_{db} \\
Wait_{db} &= \underbrace{ack_b . Sender_{1-b}}_{\text{successful}} + \underbrace{ack_{1-b}.Send_{db} + ack_\perp . Send_{db}}_{\text{error}}
\end{aligned}
$$

Repetition: Implementation of Receiver

Receiver gets frames of the form db or $\bot$. In the first case, if b has the expected value, d is forwarded via $deliver_d$, and b is returned via Ack. Otherwise the transmission is re-initiated by returning the "wrong" control bit $1 - b$ to $Sender$.

Formally, for $b \in \{0, 1\}$ and $d \in D$:

$$\begin{array}{rcl} Receiver & = & Receiver_0 \\ Receiver_b & = & \sum_{d \in D} trans_{db}.Reply_{db} \\ & + & \sum_{d \in D} trans_{d(1-b)}.\overline{reply_{1-b}}.Receiver_b \\ & + & trans_{\bot}.\overline{reply_{1-b}}Receiver_b \\ Reply_{db} & = & \overline{deliver_d}.\overline{reply_b}.Receiver_{1-b} \end{array}$$

Repetition: Correctness of ABP I

Theorem

$ABP(\overrightarrow{accept}, \overrightarrow{deliver}) \simeq Buffer(\overrightarrow{accept}, \overrightarrow{deliver})$

Remark: because of internal τ-steps in ABP, $ABP \sim Buffer$ cannot hold.

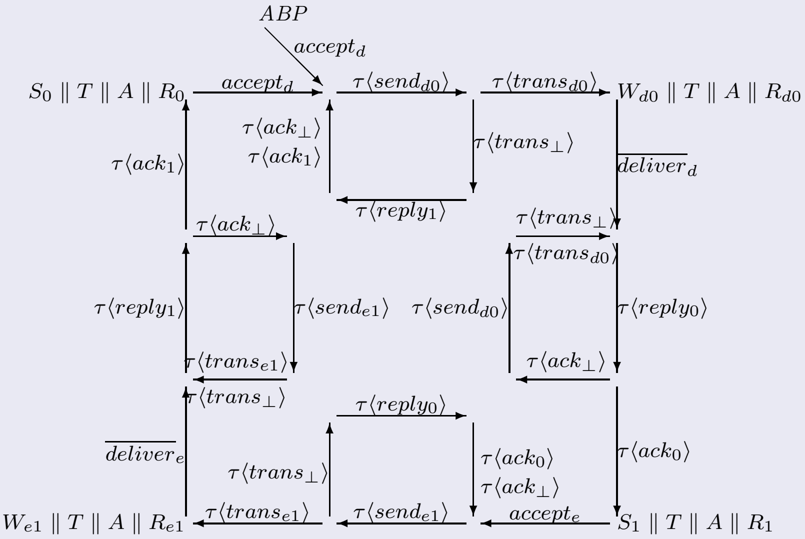
Proof.

1. Construct transition system of $ABP(\overrightarrow{accept}, \overrightarrow{deliver})$ (next slide; $S = Sender$, $W = Wait$, $T = Trans$, $A = Ack$, $R = Receiver/Reply$, $d, e \in D$; without restrictions)
2. Show that $ABP(\overrightarrow{accept}, \overrightarrow{deliver}) \approx Buffer(\overrightarrow{accept}, \overrightarrow{deliver})$
3. $ABP(\overrightarrow{accept}, \overrightarrow{deliver}) \not\xrightarrow{\tau}$ and $Buffer(\overrightarrow{accept}, \overrightarrow{deliver}) \not\xrightarrow{\tau}$ $\implies ABP(\overrightarrow{accept}, \overrightarrow{deliver}) \simeq Buffer(\overrightarrow{accept}, \overrightarrow{deliver})$

□

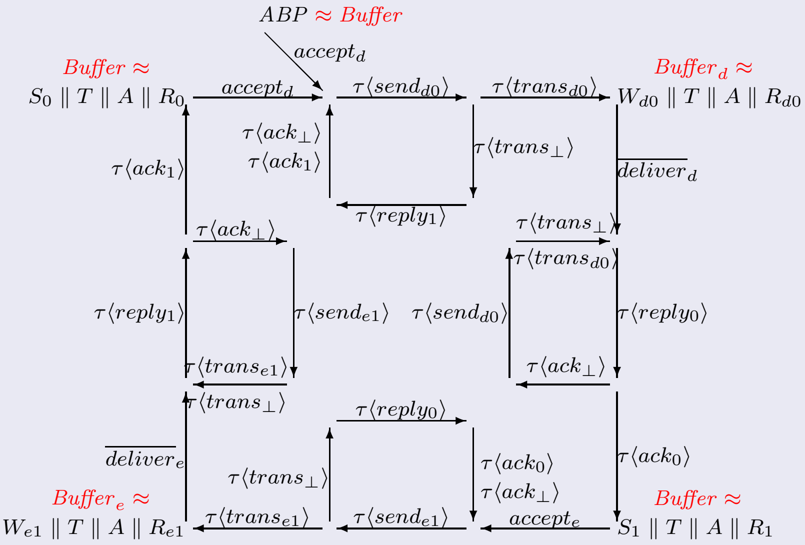
Repetition: Correctness of ABP II

Proof (continued).



Repetition: Correctness of ABP II

Proof (continued).



Outline

Duplication of Messages I

Duplication of messages can be modelled as follows:

$$\begin{aligned} Trans &= \sum_{f \in F} send_f.(\underbrace{\overline{trans_f}.Trans}_{\text{successful}} + \\ &\qquad \underbrace{\overline{trans_\perp}.Trans}_{\text{error}} + \\ &\qquad \underbrace{\overline{trans_f}.\overline{trans_f}.Trans}_{\text{duplication}}) \end{aligned}$$

$$Ack = \sum_{b \in \{0,1\}} reply_b.(\underbrace{\overline{ack_b}.Ack}_{\text{successful}} + \underbrace{\overline{ack_\perp}.Ack}_{\text{error}} + \underbrace{\overline{ack_b}.\overline{ack_b}.Ack}_{\text{duplication}})$$

Duplication of Messages I

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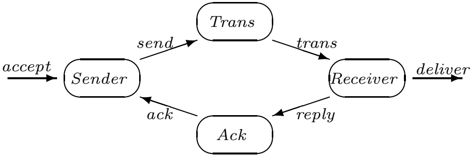
$$Ack = \sum_{b \in \{0,1\}} reply_b.(\underbrace{\overline{ack_b}.Ack}_{\text{successful}} + \underbrace{\overline{ack_\perp}.Ack}_{\text{error}} + \underbrace{\overline{ack_b}.\overline{ack_b}.Ack}_{\text{duplication}})$$

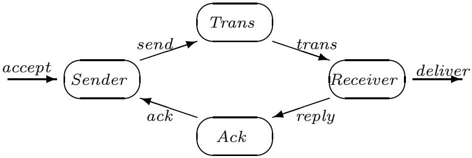
Duplication of Messages II



Now the ABP behaves as follows (without restriction):

$$Sender_0 \parallel Trans \parallel Ack \parallel Receiver_0$$

Duplication of Messages II



Now the ABP behaves as follows (without restriction):

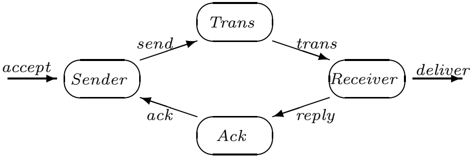
$$Sender_b = \sum_{d \in D} accept_d . Send_{db}$$

$$Sender_0 \parallel Trans \parallel Ack \parallel Receiver_0$$

$$\downarrow accept_d$$

$$Send_{d0} \parallel Trans \parallel Ack \parallel Receiver_0$$

Duplication of Messages II



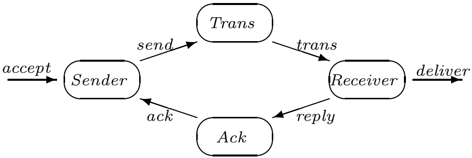
Now the ABP behaves as follows (without restriction):

$$Send_{db} = \textcolor{red}{\overline{send_{db}}}.Wait_{db}$$

$$Trans = \sum_{f \in F} \textcolor{red}{send_f}.(\overline{trans_f}.Trans + \overline{trans_\perp}.Trans + \overline{trans_f}.\overline{trans_f}.Trans)$$

$$Send_{d0} \parallel Trans \parallel Ack \parallel Receiver_0$$
$$\downarrow \tau\langle send_{d0} \rangle$$
$$Wait_{d0} \parallel (\ldots + \overline{trans_{d0}}.\overline{trans_{d0}}.Trans) \parallel Ack \parallel Receiver_0$$

Duplication of Messages II



Now the ABP behaves as follows (without restriction):

$$Receiver_b = \sum_{d \in D} trans_{db}.Reply_{db} + \ldots$$

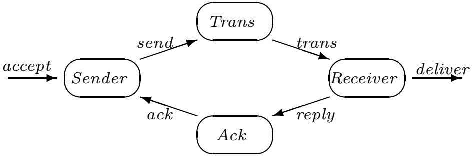
$$Trans = \sum_{f \in F} send_f.(\overline{trans_f}.Trans + \overline{trans_\perp}.Trans + \overline{trans_f}.\overline{trans_f}.Trans)$$

$$Wait_{d0} \parallel (\ldots + \overline{trans_{d0}}.\overline{trans_{d0}}.Trans) \parallel Ack \parallel Receiver_0$$

$$\downarrow \tau \langle trans_{d0} \rangle$$

$$Wait_{d0} \parallel \overline{trans_{d0}}.Trans \parallel Ack \parallel Reply_{d0}$$

Duplication of Messages II

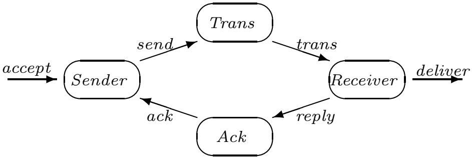


Now the ABP behaves as follows (without restriction):

$$Reply_{db} = \overline{deliver_d}.\overline{reply_b}.Receiver_{1-b}$$

$$Wait_{d0} \parallel \overline{trans_{d0}}.Trans \parallel Ack \parallel Reply_{d0}$$
$$\downarrow \overline{deliver_d}$$
$$Wait_{d0} \parallel \overline{trans_{d0}}.Trans \parallel Ack \parallel \overline{reply_0}.Receiver_1$$

Duplication of Messages II



Now the ABP behaves as follows (without restriction):

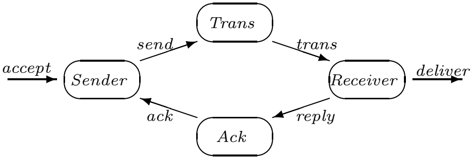
$$Reply_{db} = \overline{deliver_d}.\overline{reply_b}.Receiver_{1-b}$$

$$Ack = \sum_{b\in\{0,1\}} reply_b.(\overline{ack_b}.Ack + \overline{ack_\perp}.Ack + \overline{ack_b}.\overline{ack_b}.Ack)$$

$$Wait_{d0} \parallel \overline{trans_{d0}}.Trans \parallel Ack \parallel \overline{reply_0}.Receiver_1$$

$$\downarrow \tau\langle reply_0\rangle$$

$$Wait_{d0} \parallel \overline{trans_{d0}}.Trans \parallel (\ldots + \overline{ack_0}.\overline{ack_0}.Ack) \parallel Receiver_1$$



Now the ABP behaves as follows (without restriction):

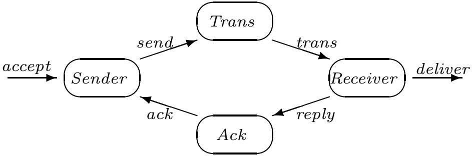
$$Wait_{db} = ack_b.Sender_{1-b} + ack_{1-b}.Send_{db} + ack_{\perp}.Send_{db}$$

$$Ack = \sum_{b \in \{0,1\}} reply_b.(\overline{ack_b}.Ack + \overline{ack_{\perp}}.Ack + \overline{ack_b}.\overline{ack_b}.Ack)$$

$$Wait_{d0} \parallel \overline{trans_{d0}}.Trans \parallel (\ldots + \overline{ack_0}.\overline{ack_0}.Ack) \parallel Receiver_1$$

$$\downarrow \tau\langle ack_0 \rangle$$

$$Sender_1 \parallel \overline{trans_{d0}}.Trans \parallel \overline{ack_0}.Ack \parallel Receiver_1$$



Now the ABP behaves as follows (without restriction):

$$Receiver_b = \ldots + \sum_{d \in D} trans_{d(1-b)}.\overline{reply_{1-b}}.Receiver_b$$

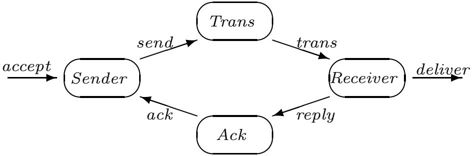
$$Trans = \sum_{f \in F} send_f.(\overline{trans_f}.Trans + \overline{trans_\perp}.Trans + \overline{trans_f}.\overline{trans_f}.Trans)$$

$$Sender_1 \parallel \overline{trans_{d0}}.Trans \parallel \overline{ack_0}.Ack \parallel Receiver_1$$

$$\downarrow \tau\langle trans_{d0}\rangle$$

$$Sender_1 \parallel Trans \parallel \overline{ack_0}.Ack \parallel \overline{reply_0}.Receiver_1$$

Duplication of Messages II



Now the ABP behaves as follows (without restriction):

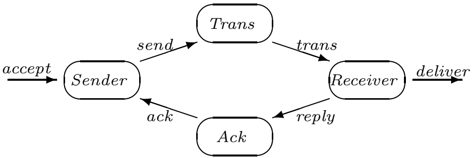
$$Sender_b = \sum_{d \in D} accept_d . Send_{db}$$

$$Sender_1 \parallel Trans \parallel \overline{ack_0}.Ack \parallel \overline{reply_0}.Receiver_1$$

$$\downarrow accept_e$$

$$Send_{e1} \parallel Trans \parallel \overline{ack_0}.Ack \parallel \overline{reply_0}.Receiver_1$$

Duplication of Messages II



Now the ABP behaves as follows (without restriction):

$$Send_{db} = \overline{send_{db}}.Wait_{db}$$

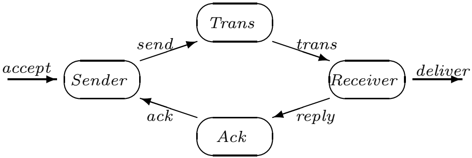
$$Trans = \sum_{f \in F} send_f.(\overline{trans_f}.Trans + \overline{trans_\bot}.Trans + \overline{trans_f}.\overline{trans_f}.Trans)$$

$$Send_{e1} \parallel Trans \parallel \overline{ack_0}.Ack \parallel \overline{reply_0}.Receiver_1$$

$$\downarrow \tau \langle send_{e1} \rangle$$

$$Wait_{e1} \parallel (\ldots + \overline{trans_{e1}}.\overline{trans_{e1}}.Trans) \parallel \overline{ack_0}.Ack \parallel \overline{reply_0}.Receiver_1$$

Duplication of Messages II



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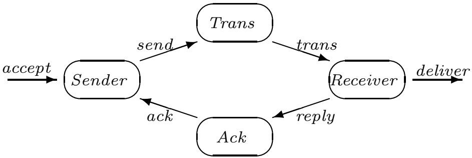
$$Wait_{db} = ack_b.Sender_{1-b} + ack_{1-b}.Send_{db} + ack_{\perp}.Send_{db}$$

$$Ack = \sum_{b \in \{0,1\}} reply_b.(\overline{ack_b}.Ack + \overline{ack_{\perp}}.Ack + \overline{ack_b}.\overline{ack_b}.Ack)$$

$$Wait_{e1} \parallel (\ldots + \overline{trans_{e1}}.\overline{trans_{e1}}.Trans) \parallel \overline{ack_0}.Ack \parallel \overline{reply_0}.Receiver_1$$

$$\downarrow \tau\langle ack_0 \rangle$$

$$Send_{e1} \parallel (\ldots + \overline{trans_{e1}}.\overline{trans_{e1}}.Trans) \parallel Ack \parallel \overline{reply_0}.Receiver_1$$



Now the ABP behaves as follows (without restriction):

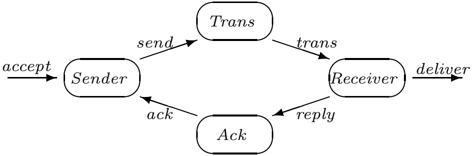
$$Receiver_b = \ldots + \sum_{d \in D} trans_{d(1-b)}.\overline{reply_{1-b}}.Receiver_b$$

$$Ack = \sum_{b \in \{0,1\}} reply_b.(\overline{ack_b}.Ack + \overline{ack_\perp}.Ack + \overline{ack_b}.\overline{ack_b}.Ack)$$

$$Send_{e1} \parallel (\ldots + \overline{trans_{e1}}.\overline{trans_{e1}}.Trans) \parallel Ack \parallel \overline{reply_0}.Receiver_1$$

$$\downarrow \tau\langle reply_0 \rangle$$

$$Send_{e1} \parallel (\ldots + \overline{trans_{e1}}.\overline{trans_{e1}}.Trans) \parallel (\ldots + \overline{ack_0}.\overline{ack_0}.Ack) \parallel Receiver$$



Now the ABP behaves as follows (without restriction):

$$Receiver_b = \sum_{d \in D} trans_{db}.Reply_{db} + \ldots$$

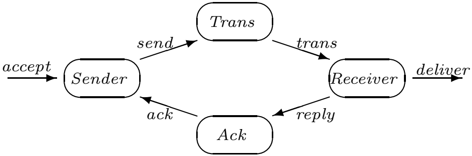
$$Trans = \sum_{f \in F} send_f.(\overline{trans_f}.Trans + \overline{trans_\perp}.Trans + \overline{trans_f}.\overline{trans_f}.Trans)$$

$$Send_{e1} \parallel (\ldots + \overline{trans_{e1}}.\overline{trans_{e1}}.Trans) \parallel (\ldots + \overline{ack_0}.\overline{ack_0}.Ack) \parallel Receiver$$

$$\downarrow \tau \langle trans_{e1} \rangle$$

$$Send_{e1} \parallel \overline{trans_{e1}}.Trans \parallel (\ldots + \overline{ack_0}.\overline{ack_0}.Ack) \parallel Reply_{e1}$$

Duplication of Messages II



Now the ABP behaves as follows (without restriction):

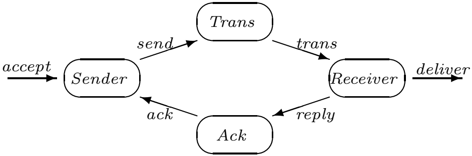
$$Reply_{db} = \overline{deliver_d}.\overline{reply_b}.Receiver_{1-b}$$

$$Send_{e1} \parallel \overline{trans_{e1}}.Trans \parallel (\ldots + \overline{ack_0}.\overline{ack_0}.Ack) \parallel Reply_{e1}$$

$$\downarrow \overline{deliver_e}$$

$$Send_{e1} \parallel \overline{trans_{e1}}.Trans \parallel (\ldots + \overline{ack_0}.\overline{ack_0}.Ack) \parallel \overline{reply_1}.Receiver_0$$

Duplication of Messages II



Now the ABP behaves as follows (without restriction):

$$Send_{e1} \parallel \overline{trans_{e1}}.Trans \parallel (\ldots + \overline{ack_0}.\overline{ack_0}.Ack) \parallel \overline{reply_1}.Receiver_0$$

$$\downarrow$$

$$?$$

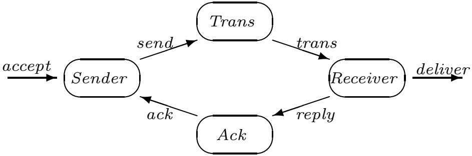
Deadlock $\Longrightarrow$ ABP cannot handle this

Outline

Handling Duplication of Messages

- **Idea:** allow *Sender* and *Receiver* to transmit $\perp$ frames:
  - $Receiver \xrightarrow{reply_\perp}$: message not received
  - $Sender \xrightarrow{send_\perp}$: acknowledgment not received
- Allows to distinguish corrupted and duplicated frames

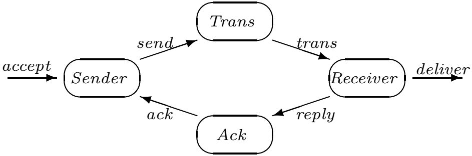
Modified Implementation of Sender



For $b \in \{0,1\}$ and $d \in D$:

$$
\begin{aligned}
Sender &= Sender_0\\
Sender_b &= \sum_{d \in D} accept_d . Send_{db}\\
Send_{db} &= \overline{send_{db}} . Wait_{db}\\
Wait_{db} &= \underbrace{ack_b . Sender_{1-b}}_{\text{successful}} + \underbrace{ack_\perp . Send_{db}}_{\text{error, restart}} + \underbrace{ack_{1-b} . Wait_{db}}_{\text{duplication, ignore}}
\end{aligned}
$$

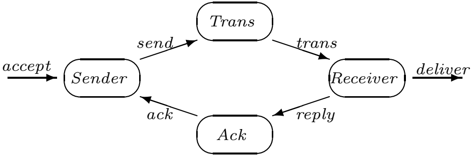
Modified Implementation of Sender



For $b \in \{0, 1\}$ and $d \in D$:

$$\begin{aligned} Sender &= Sender_0 \\ Sender_b &= \sum_{d \in D} accept_d.Send_{db} \\ Send_{db} &= \overline{send_{db}}.Wait_{db} \\ Wait_{db} &= \underbrace{ack_b.Sender_{1-b}}_{\text{successful}} + \underbrace{ack_\perp.Send_{db}}_{\text{error, restart}} + \underbrace{ack_{1-b}.Wait_{db}}_{\text{duplication, ignore}} \end{aligned}$$

Modified Implementation of Sender



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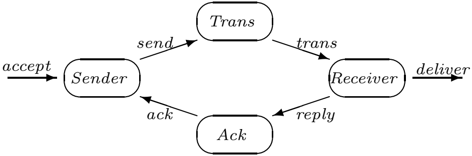
\begin{aligned}
Sender &= Sender_0 \\
Sender_b &= \sum_{d \in D} accept_d . Send_{db} \\
Send_{db} &= \overline{send_{db}} . Wait_{db} \\
Wait_{db} &= \underbrace{ack_b . Sender_{1-b}}_{\text{successful}} + \underbrace{ack_\perp . Send_{db}}_{\text{error, restart}} + \underbrace{ack_{1-b} . Wait_{db}}_{\text{duplication, ignore}}
\end{aligned}
$$

Modified Implementation of Sender



For $b \in \{0,1\}$ and $d \in D$:

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\begin{aligned}
Sender &= Sender_0 \\
Sender_b &= \sum_{d \in D} accept_d.Send_{db} \\
Send_{db} &= \overline{send_{db}}.Wait_{db} \\
Wait_{db} &= \underbrace{ack_b.Sender_{1-b}}_{\text{successful}} + \underbrace{ack_\perp.Send_{db}}_{\text{error, restart}} + \underbrace{\color{red}{ack_{1-b}.Wait_{db}}}_{\text{duplication, ignore}}
\end{aligned}
$$

Modified Implementation of Receiver

For $b \in \{0, 1\}$ and $d \in D$:

$$
\begin{aligned}
Receiver &= Receiver_0 \\
Receiver_b &= \sum_{d \in D} trans_{db}.Reply_{db} \\
&+ trans_{\perp}.\overline{reply_{\perp}}.Receiver_b \\
&+ \sum_{d \in D} trans_{d(1-b)}.Receiver_b \\
Reply_{db} &= \overline{deliver_d}.\overline{reply_b}.Receiver_{1-b}
\end{aligned}
$$

For $b \in \{0,1\}$ and $d \in D$:

$$\begin{aligned}
Receiver &= Receiver_0 \\
Receiver_b &= \sum_{d \in D} trans_{db}.Reply_{db} \\
&+ trans_{\bot}.\overline{reply_{\bot}}.Receiver_b \\
&+ \sum_{d \in D} trans_{d(1-b)}.Receiver_b \\
Reply_{db} &= \overline{deliver_d}.\overline{reply_b}.Receiver_{1-b}
\end{aligned}$$

Modified Implementation of Receiver

For $b \in \{0,1\}$ and $d \in D$:

$$
\begin{aligned}
\mathit{Receiver} &= \mathit{Receiver}_0 \\
\mathit{Receiver}_b &= \sum_{d \in D} \mathit{trans}_{db}.\mathit{Reply}_{db} \\
&+ \mathit{trans}_{\bot}.\overline{\mathit{reply}_{\bot}}.\mathit{Receiver}_b \\
&+ \sum_{d \in D} \mathit{trans}_{d(1-b)}.\mathit{Receiver}_b \\
\mathit{Reply}_{db} &= \overline{\mathit{deliver}_d}.\overline{\mathit{reply}_b}.\mathit{Receiver}_{1-b}
\end{aligned}
$$

The Overall System

$$
\begin{aligned}
& ABP(\overrightarrow{accept}, \overrightarrow{deliver}) \\
& \quad = \mathsf{new}\, L\, (Sender \parallel Trans \parallel Ack \parallel Receiver)
\end{aligned}
$$

$$
\begin{aligned}
Sender &= Sender_0 \\
Sender_b &= \textstyle\sum_{d \in D} accept_d . Send_{db} \\
Send_{db} &= \overline{send_{db}} . Wait_{db} \\
Wait_{db} &= ack_b . Sender_{1-b} + ack_\perp . Send_{db} + ack_{1-b} . Wait_{db}
\end{aligned}
$$

$$
\begin{aligned}
Receiver &= Receiver_0 \\
Receiver_b &= \textstyle\sum_{d \in D} trans_{db} . Reply_{db} \\
&\quad + trans_\perp . \overline{reply_\perp} . Receiver_b \\
&\quad + \textstyle\sum_{d \in D} trans_{d(1-b)} . Receiver_b \\
Reply_{db} &= \overline{deliver_d} . \overline{reply_b} . Receiver_{1-b}
\end{aligned}
$$

$$
\begin{aligned}
Trans &= \textstyle\sum_{f \in F} send_f . (\overline{trans_f} . Trans + \overline{trans_\perp} . Trans + \\
&\qquad\qquad\qquad \overline{trans_f} . \overline{trans_f} . Trans) \\
Ack &= \textstyle\sum_{b \in \{0,1\}} reply_b . (\overline{ack_b} . Ack + \overline{ack_\perp} . Ack + \overline{ack_b} . \overline{ack_b} . Ack)
\end{aligned}
$$

where $L := \{send_{db}, trans_{db}, reply_b, ack_b \mid db \in F\}$
$\cup\ \{send_\perp, trans_\perp, reply_\perp, ack_\perp\}$

Correctness of the Modification

Again:

Theorem 11.1

$ABP(\overrightarrow{accept}, \overrightarrow{deliver}) \simeq Buffer(\overrightarrow{accept}, \overrightarrow{deliver})$

Proof.

on the board
($S = Sender/Send$, $W = Wait$, $T = Trans$, $A = Ack$,
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- Handling loss of messages: by introducing timeouts
- Validity of correctness proof (τ-cycles in ABP, but not in *Buffer*)?

  Simplest case:

  $$A(a) = \tau.A + a.\mathsf{nil} \quad \simeq \quad B(a) = \tau.a.\mathsf{nil}$$

  Even more: every LTS containing τ-cycles is observationally congruent to one without τ-cycles
- There are notions of equivalence which distinguish divergent (τ-cycles) and convergent (no τ-cycles) processes
- **But:**
  - they are more complicated than standard bisimulation
  - (weak) bisimulation allows the proportion between the speeds of processes to vary unboundedly – why not infinite?
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Mobility in Concurrent Systems I

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$\Longrightarrow$ every potential communication partner known beforehand, no dynamic passing of communication links

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Example 11.2 (Dynamic access to resources)

- Server S controls access to printer P
- Client C wishes to use P
- In CCS: P and C must share some action name a
  $\Longrightarrow$ C could access P without being granted it by S
- In π-calculus :
  - initially only S has access to P (using link a)
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- Formally:

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$$\xrightarrow{\tau} S' \parallel C' \parallel P'[e \mapsto d]$$

- a: link to P
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