

Modeling Concurrent and Probabilistic Systems

Lecture 6: Properties of Strong Bisimulation

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Summer Semester 2009

- 1 Repetition: Definition of Strong Bisimulation
- 2 Congruence Property of Strong Bisimulation
- 3 Deadlock Sensitivity of Strong Bisimulation
- 4 Traces and Deadlocks
- 5 Decidability of Strong Bisimulation
- 6 Summary

Repetition: Definition of Strong Bisimulation

Definition (Strong bisimulation)

A relation $\rho \subseteq Prc \times Prc$ is called a **strong bisimulation** if $P\rho Q$ implies, for every $\alpha \in Act$,

$$\textcircled{1} \quad P \xrightarrow{\alpha} P' \implies \text{ex. } Q' \in Prc \text{ such that } Q \xrightarrow{\alpha} Q' \text{ and } P'\rho Q'$$

$$\textcircled{2} \quad Q \xrightarrow{\alpha} Q' \implies \text{ex. } P' \in Prc \text{ such that } P \xrightarrow{\alpha} P' \text{ and } P'\rho Q'$$

$P, Q \in Prc$ are called **strongly bisimilar** (notation: $P \sim Q$) if there exists a strong bisimulation ρ such that $P\rho Q$.

Theorem

\sim is an equivalence relation.

Proof.

on the board



Example

(on the board)

1

$$\begin{array}{ccc} P & \sim & Q_1 \\ \circlearrowleft & & a \downarrow \uparrow a \\ a & & Q_2 \end{array}$$

2

$$\begin{array}{ccc} P & \not\sim & Q \\ \downarrow a & & a \swarrow \searrow a \\ P_1 & & Q_1 \quad Q_3 \\ b \swarrow \searrow c & & b \downarrow \quad \downarrow c \\ P_2 \quad P_3 & & Q_2 \quad Q_4 \end{array}$$

(remember: $Tr(P) = Tr(Q)$)

Repetition: Bisimulation Implies Trace Equivalence

Definition (Trace language; repetition)

The **trace language** of $P \in \text{Prc}$ is given by

$$\text{Tr}(P) := \{w \in \text{Act}^* \mid \text{ex. } P' \in \text{Prc} \text{ such that } P \xrightarrow{w} P'\}.$$

Theorem 6.4

For every $P, Q \in \text{Prc}$, $P \sim Q$ implies $\text{Tr}(P) = \text{Tr}(Q)$.

Proof.

- Assume that $P \sim Q$ but (w.l.o.g.) $w \in \text{Tr}(P) \setminus \text{Tr}(Q)$.
- Let $v \in \text{Act}^*$ be the longest prefix of w such that $v \in \text{Tr}(Q)$ (i.e., $w = v\alpha u$ for some $\alpha \in \text{Act}$ and $u \in \text{Act}^*$).
- Let $P', P'' \in \text{Prc}$ such that $P \xrightarrow{v} P' \xrightarrow{\alpha} P''$.
- Since $P \sim Q$ there exists $Q' \in \text{Prc}$ such that $Q \xrightarrow{v} Q'$ and $P' \sim Q'$ (by induction on $|v|$).
- But we have that $P' \xrightarrow{\alpha} P''$ whereas $Q' \not\xrightarrow{\alpha}$ \nmid

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Congruence Property of Strong Bisimulation I

The congruence proof employs the following lemma.

Lemma 6.1

For every $P, Q, R \in \text{Prc}$,

- ① $P + Q \sim Q + P$
- ② $P + (Q + R) \sim (P + Q) + R$
- ③ $P + \text{nil} \sim P$
- ④ $P \parallel Q \sim Q \parallel P$
- ⑤ $P \parallel (Q \parallel R) \sim (P \parallel Q) \parallel R$
- ⑥ $P \parallel \text{nil} \sim P$

Proof.

- ① on the board
- ② on the board
- ③ 3rd ex. sheet
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- ⑤ 3rd ex. sheet
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Proof.

- ❶ on the board
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- ❻ on the board



Congruence Property of Strong Bisimulation II

Definition (CCS congruence; repetition)

An equivalence relation $\cong \subseteq \text{Prc} \times \text{Prc}$ is said to be a **CCS congruence** if it is preserved by the CCS constructs; that is, if $P \cong Q$ then

$$\alpha.P \cong \alpha.Q$$

$$P + R \cong Q + R$$

$$R + P \cong R + Q$$

$$P \parallel R \cong Q \parallel R$$

$$R \parallel P \cong R \parallel Q$$

$$\text{new } a P \cong \text{new } a Q$$

for every $\alpha \in \text{Act}$, $R \in \text{Prc}$, and $a \in N$.

Theorem 6.2

\sim is a CCS congruence.

Proof.

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Deadlock Sensitivity of Strong Bisimulation

Definition (Deadlock; repetition)

Let $P, Q \in \text{Prc}$ and $w \in \text{Act}^*$ such that $P \xrightarrow{w} Q$ and $Q \not\rightarrow$. Then Q is called a **w-deadlock** of P .

An equivalence relation $\cong \subseteq \text{Prc} \times \text{Prc}$ is called **deadlock sensitive** if for every $P \cong Q$ such that P has a w -deadlock, Q also has a w -deadlock.

Theorem 6.3

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Proof.

on the board □

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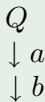
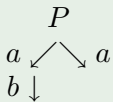


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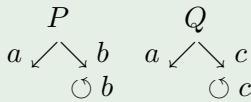
Traces and Deadlocks

Remark: traces and deadlocks are independent in the following sense

Example 6.4



same traces
different deadlocks



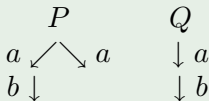
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same deadlocks

But: processes with finite trace sets and identical deadlocks are trace equivalent (since every trace is a prefix of some deadlock)

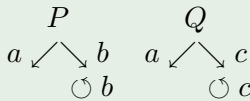
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