RWTH RHEINISCH-WESTFÄLISCHE TECHNISCHE HOCHSCHULE AACHEN

LEHRSTUHL FÜR INFORMATIK 2
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Modeling and Verification of Probabilistic Systems
Summer term 2011

– Series 1 –

Hand in on April 20th before the exercise class.

Exercise 1 (4 points)

Let Ω be a countably infinite set and define $\mathfrak{F}$ as the smallest class of subsets of Ω such that for all $A \subseteq \Omega$:

$$|A| < +\infty \Rightarrow A \in \mathfrak{F} \qquad \text{and} \qquad A \in \mathfrak{F} \Rightarrow A^c \in \mathfrak{F},$$

where $A^c = \Omega - A$ denotes the complement.

a) Show the definition is non-trivial, i.e., in general $\mathfrak{F} \neq 2^\Omega$.
(Hint: find a set Ω and a subset $A \subseteq \Omega$ which cannot be in $\mathfrak{F}$ according to the above definition.)

b) Why is it important that $\mathfrak{F}$ is defined as the *smallest* class of subset (and not e.g. the *largest*)?

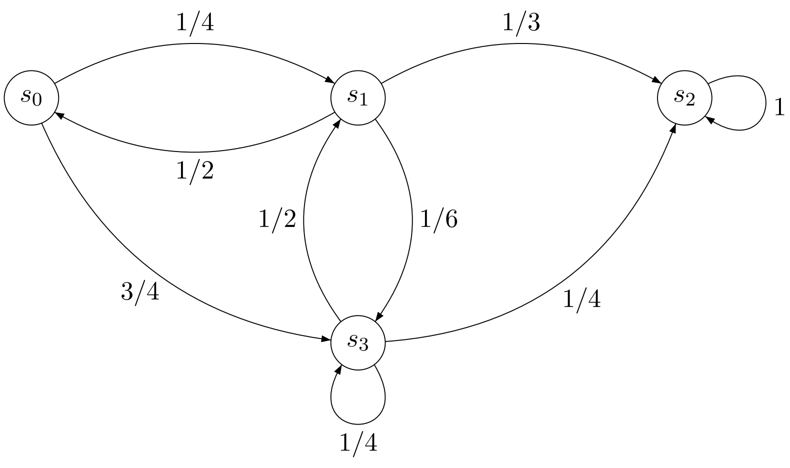
c) Demonstrate that $\mathfrak{F}$ is a σ-algebra as defined in the lecture.

Exercise 2 (3 points)

Let X be a discrete random variable with a geometric distribution, i.e., for $k > 0$, $\Pr\{X = k\} = (1-p)^{k-1} \cdot p$. Show that $E[X] = \frac{1}{p}$ and $\text{Var}[X] = \frac{1-p}{p^2}$.

Exercise 3 (3 points)

Given the DTMC $\mathcal{D}$ as follows:



a) Compute the probability of going from s_0 to s_3 in *exactly* 3 steps;
(*Hints: by the end of the 3rd step the system is in state* 3.)

b) Compute the probability of going from s_0 to s_3 in *at most* 3 steps;

(*Hints: by the end of the* 3*rd step the system has been in state* 3.)

c) Compute the probability of being in state s_2 after 3 steps when the initial distribution is uniform over all states.