RHEINISCH-WESTFÄLISCHE TECHNISCHE HOCHSCHULE AACHEN

LEHRSTUHL FÜR INFORMATIK 2
RWTH Aachen · D-52056 Aachen
http://www-i2.informatik.rwth-aachen.de/

Prof. Dr. Ir. J.-P. Katoen
F. Gretz & F. Sher

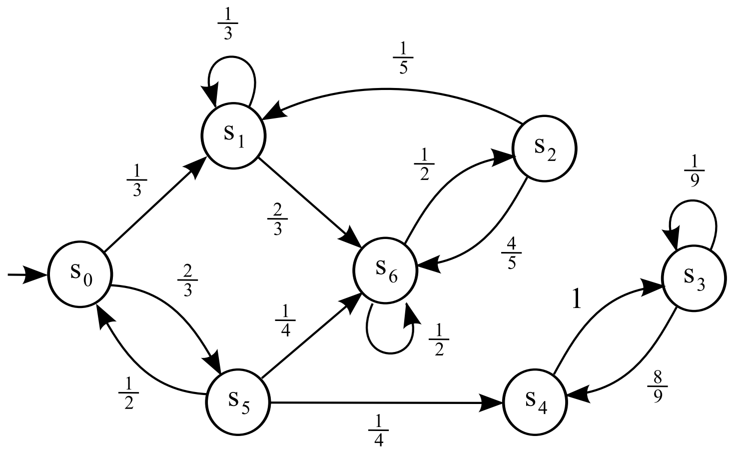
Modeling and Verification of Probabilistic Systems
Summer term 2011

– Series 2 –

Hand in on April 27th before the exercise class.

Exercise 1 (3 points)

Consider the DTMC below:



Let $A = \{s_3\}$ and $B = \{s_2\}$. *(Hint: for a),b),d) and e) you do not have to solve linear equation systems or compute powers of matrices.)*

a) Compute the probability measure of the union of the following cylinder sets: $\mathrm{Cyl}(s_0 s_1)$, $\mathrm{Cyl}(s_0 s_5 s_6)$, $\mathrm{Cyl}(s_0 s_5 s_4 s_3)$, $\mathrm{Cyl}(s_0 s_1 s_6)$

b) Compute the probability, from each state of the Markov chain, of reaching a state in A within 4 steps.

c) Compute the probability, from each state of the Markov chain, of reaching a state in A.

d) What is the probability, from the initial state, of reaching the set of states $A \cup B$?

e) What is the probability, from the initial state, that a state from $A \cup B$ is visited infinitely often?

Exercise 2 (3 points)

Consider a finite DTMC $D = (S, \mathbf{P}, s_{\text{init}}, AP, L)$ and subsets of states $A, B \subseteq S$. Show that the following two sets of paths are measurable, i.e. contained in the σ-algebra of D:

a) the set of paths starting in state s_{init} and remaining forever in states from A;

b) the set of paths starting in state s_{init}, remaining forever in states from A and passing through a state in B after exactly 5 time-steps.

Exercise 3 (4 points)

Consider the Markov Chain from Exercise 1 again. We now define new subsets of states as following:

- $C = \{s_0, s_1, s_4, s_6\}$
- $D = \{s_3, s_4\}$
- $E = \{s_2, s_3\}$

a) Compute $Pr(s_0 \models \Diamond E)$ by solving a linear equation system.

b) Compute $Pr(s_0 \models C \mathsf{U}^{\leq 5} E)$ using:

  (a) linear equation systems;

  (b) transient state probabilities.

c) Determine $Pr(s_0 \models \Diamond \Box D)$.