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Modeling and Verification of Probabilistic Systems
Summer term 2011

– Series 3 –

Hand in on May 4th before the exercise class.

Exercise 1 (4 points)

Let $M = (S, \mathbf{P}, \iota_{init}, AP, L)$ be a finite Markov chain, $s \in S$ and $C, B \subseteq S$, $n \in \mathbb{N}$, $n \geq 1$. Let $C \mathsf{U}^{=n} B$ denote the event that a B-state will be entered after exactly n steps and all states that are visited before belong to C. That is, $s_0\, s_1\, s_2 \ldots \models C \mathsf{U}^{=n} B$ if and only if $s_n \in B$ and $s_i \in C$ for $0 \leq i < n$. The event $C \mathsf{U}^{\geq n} B$ denotes the union of the events $C \mathsf{U}^{=k} B$ where k ranges over all natural numbers $\geq n$.

Question: Provide an algorithm to compute

(a) the values $\Pr(s \models C \mathsf{U}^{=n} B)$, and

(b) an algorithm for computing $\Pr(s \models C \mathsf{U}^{\geq n} B)$.

Exercise 2 (3 points)

In this exercise we compare the computation of probabilities of qualitative properties of a DTMC to model checking against CTL (i.e. discarding all probability labels and treating the Markov chain as a transition system). Let $M = (S, \mathbf{P}, \iota_{init}, AP, L)$ be a finite Markov chain and $s \in S$, a, $b \in AP$. Prove or disprove the following statements:

(a) $\Pr(s \models \Box a) = 1$ iff $s \models \forall \Box a$

(b) $\Pr(s \models \Diamond a) < 1$ iff $s \not\models \forall \Diamond a$

(c) $\Pr(s \models \Box a) > 0$ iff $s \models \exists \Box a$

(d) $\Pr(s \models \Diamond \Box a) = 1$ iff $\Pr(s \models \Diamond B) = 1$ where $B = \{t \in S \mid t \models \forall \Box a\}$

(e) $\Pr(s \models a \mathsf{U} b) = 1$ iff $s \models \forall (a \mathsf{U} b)$

(f) $\Pr(s \models a \mathsf{U} b) = 0$ iff $s \not\models \exists (a \mathsf{U} b)$

Exercise 3 (3 points)

Consider the following simple game between 2 players who, between them, have n coins. Initially, player 1 has m of these coins. In each turn of the game, both players simultaneously toss one of their coins. If the two coins are the same, player 1 keeps both coins; if they differ, player 2 keeps them. The game ends when one player has all the coins and is declared the winner.

a) Draw a DTMC to represent the evolution of this game.

b) What is the probability of the game terminating?

c) Assume we wish to establish if "player 1 has a better chance of winning than player 2". Express this statement using PCTL.

d) Express the following statement in PCTL: "with probability at most 0.1, player 1 will, within 5 more turns, be in a position where he has a chance to win the game in the next turn".