Modeling and Verification of Probabilistic Systems
Summer term 2011

– Series 4 –

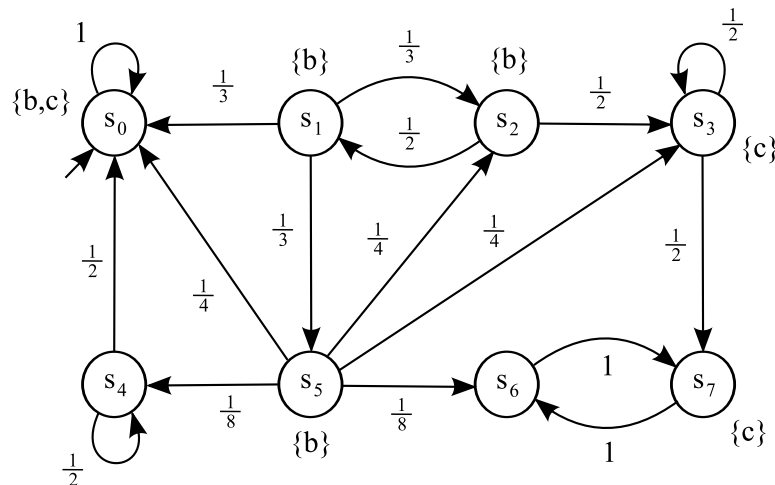Hand in on May 11th before the exercise class.

Exercise 1 (4 points)

Consider the DTMC below. Illustrate the execution of the PCTL model checking algorithms to determine which states of the Markov chain satisfy:

a) $P_{\geq \frac{17}{19}}[\, b \, U \, c\,]$

b) $P_{\geq \frac{1}{2}}[\bigcirc P_{>\frac{1}{3}}[\,(b \vee c) \, U^{\leq 2} \,(b \wedge c)\,]\,]$

Exercise 2 (3 points)

Below are definitions for two temporal (path) operators, W and $U^{[l,u]}$, which could be added to the temporal logic PCTL. Provide algorithms to perform model checking for each of these operators on any finite DTMC.

a) $\quad \omega \models \varphi_1 W \varphi_2 \quad \Leftrightarrow \quad \exists k \geq 0$ such that $\omega(k) \models \varphi_2$ and $\omega(i) \models \varphi_1$ for all $0 \leq i \leq k$
or $\omega(i) \models \varphi_1$ for all $i \geq 0$

b) $\quad \omega \models \varphi_1 U^{[l,u]} \varphi_2 \quad \Leftrightarrow \quad \exists l \leq i \leq u$ such that $\omega(i) \models \varphi_2$ and $\forall j < i, \; \omega(j) \models \varphi_1$

(where ω is an infinite path, φ_1 and φ_2 are PCTL state formulae and l, u are non-negative integers such that $l \leq u$).

Exercise 3 (3 points)

Prove or disprove the following *PCTL* equivalences:

(a) $P_{=1}(\bigcirc P_{=1}(\square a)) \equiv P_{=1}(\square P_{=1}(\bigcirc a))$

(b) $P_{>0.5}(\bigcirc P_{>0.5}(\Diamond a)) \equiv P_{>0.5}(\Diamond P_{>0.5}(\bigcirc a))$

(c) $P_{=1}(\bigcirc P_{=1}(\Diamond a)) \equiv P_{=1}(\Diamond P_{=1}(\bigcirc a))$