RHEINISCH-WESTFÄLISCHE TECHNISCHE HOCHSCHULE AACHEN

LEHRSTUHL FÜR INFORMATIK 2
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Modeling and Verification of Probabilistic Systems
Summer term 2011

– Series 5 –

Hand in on May 18th before the exercise class.

Exercise 1 (2 points)

In the lecture two definitions of strong bisimulation were given.

Definition 1
Let $TS = (S, Act, \rightarrow, I_0, AP, L)$ be a transition system and $R \subset S \times S$. Then R is a strong bisimulation on TS whenever for all $(s,t) \in R$:

1. $L(s) = L(t)$
2. if $s \xrightarrow{\alpha} s'$ then there exists $t' \in S$ such that $t \xrightarrow{\alpha} t'$ and $(s', t') \in R$
3. if $t \xrightarrow{\alpha} t'$ then there exists $s' \in S$ such that $s \xrightarrow{\alpha} s'$ and $(s', t') \in R$

Definition 2
Let $TS = (S, Act, \rightarrow, I_0, AP, L)$ and R an equivalence relation on S. Then: R is a strong bisimulation on S if for $(s,t) \in R$:

1. $L(s) = L(t)$, and
2. $P(s, \alpha, C) = P(t, \alpha, C)$ for all $C \in S/R$ and $\alpha \in Act$.

where

$$P(s, \alpha, S') = \begin{cases} 1 & \text{if } \exists s' \in S'.\ s \xrightarrow{\alpha} s' \\ 0 & \text{otherwise} \end{cases}$$

Prove that for a given equivalence relation R the two definitions are equivalent.

Exercise 2 (2 points)

Remember the definition of probabilistic bisimulation:

Let $D = (S, P, \iota_{init}, AP, L)$ be a DTMC and $R \subset S \times S$ an equivalence. Then: R is a probabilistic bisimulation on S if for any $(s,t) \in R$:

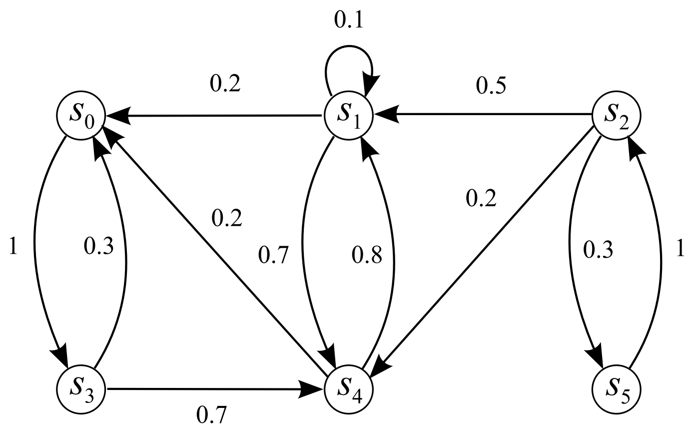
1. $L(s) = L(t)$, and
2. $P(s, C) = P(t, C)$ for all equivalence classes $C \in S/R$

where $P(s, C) = \sum_{s' \in C} P(s, s')$.

Disprove that the union of two probabilistic bisimulations is again a probabilistic bisimulation!

Exercise 3 (3 points)

Below is a 6-state DTMC. Since this Markov chain has no state labellings, applying bisimulation minimisation would show that it is bisimilar to a DTMC comprising a single state with a self-loop. Give a labelling function for the Markov chain, using the single atomic proposition b, such that bisimulation minimisation produces a bisimilar DTMC with more than 1 state (but less than 6 states). State the equivalence classes, show the resulting quotient DTMC and, for each pair of equivalence classes, give a PCTL formula that distinguishes them.



Exercise 4 (3 points)

Given the DTMC D below, give the transition system $D/\sim_p$. Show that your $\sim_p$ is indeed the coarsest bisimulation for D.