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Modeling and Verification of Probabilistic Systems
Summer term 2011

– Series 6 –

Hand in on May 25th before the exercise class.

Exercise 1 (1 points)

You have learned in the lecture that non-deterministic Büchi automata are more expressive than deterministic Büchi automata. One might wonder why the "powerset construction", which is used to compute a deterministic finite automaton from a non-deterministic one, fails for Büchi automata.

To understand this

a) construct a Büchi automaton for the language $(a+b)^*a^\omega$,

b) apply the powerset construction to this automaton and

c) show that the resulting automaton does not accept the same language.

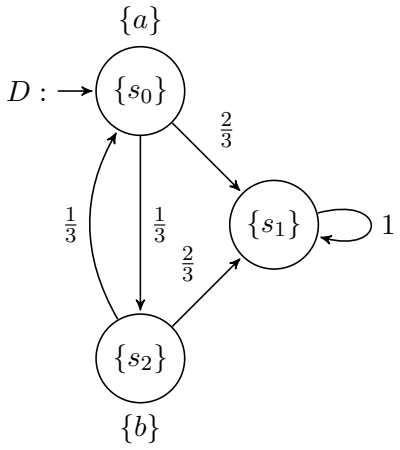
Exercise 2 (2 points)

Non-probabilistic transition systems can be model checked against non-deterministic automata. However, for DTMCs only deterministic automata are admitted. Identify the need for determinism. For this purpose,

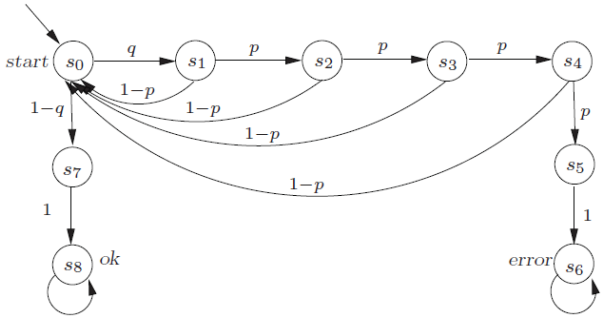
a) define a cross-product between a DTMC and an NBA,

b) construct the product of D (depicted below) and A (solution of Exercise 1 a) according to your definition and

c) comment on the result; explain what problems arise due to the non-determinism in the NBA.



Exercise 3 (3 points)



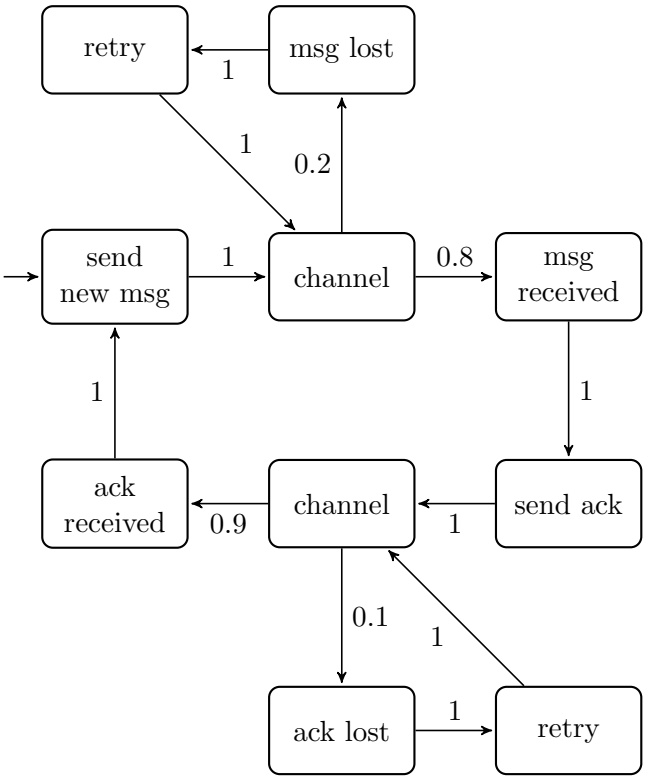
Markov chain of the IPv4 zeroconf protocol (for $n = 4$ probes).

The DTMC above models a protocol which is used to (randomly) select an unused IPv4 address within a network. The details can be found on page 751 in the book "Principles of Model Checking" by Baier and Katoen. (To solve the exercise you only need to know that p and q are probabilities.)

Task:

Given the safety property "the error state is not reached", compute the probability that this safety property is fulfilled. (Apply the algorithm for verifying regular safety properties given in the lecture.)

Exercise 4 (4 points)



Above is a DTMC D that represents a simple communication protocol. Initially a sender sends a new message via a lossy channel. With probability 0.8 the message is delivered but with probability 0.2 it is lost and after a certain timeout the sender decides to retransmit the message. Similarly the receiver should send an acknowledgement signal back to the sender after receiving the message. As before, this signal might be lost and retransmitted. Here the probability for a loss is 0.1. After the acknowledgement arrives at the sender, a new communication round is started.

The system designer claims that D satisfies the specification: "The probability that a message or an acknowledgement signal is lost at least once within one communication round is (strictly) less than 50%."

Tasks:

a) In order to reason about D and the specification, introduce the necessary labels in D.

b) The specification is of the from $\mathbb{P}_{<0.5}(\varphi)$. Formalise φ as a linear time property.

c) Construct a DBA for φ.

d) Execute the model checking algorithm for DTMC against DBA specifications in order to find out whether the claim is true or not.