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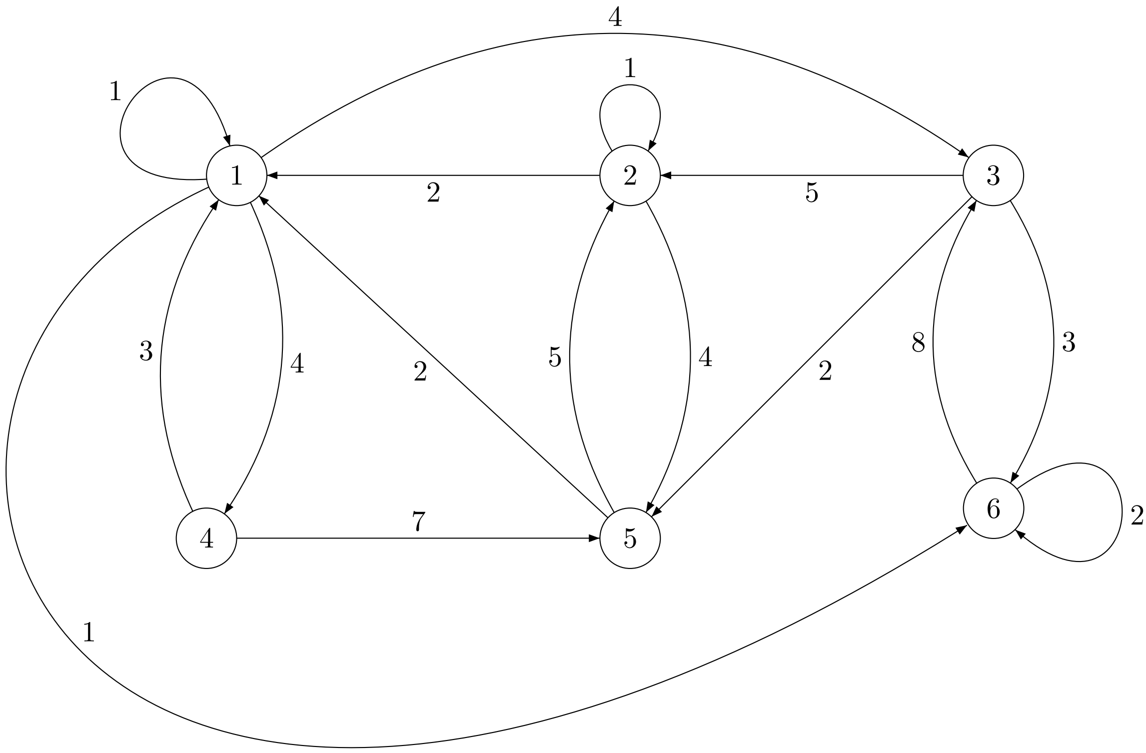
Modeling and Verification of Probabilistic Systems
Summer term 2011

– Series 10 –

Hand in on 29th June before the exercise class.



Exercise 1 **(3 points)**

Consider the CTMC given in the figure with starting distribution $p(0) = (0.1, 0.2, 0, 0, 0.4, 0.3)$.

a) Determine $C/\sim m$.

b) Determine the steady-state (limiting) distribution of C. The steady-state-distribution of a CTMC is given as:

$$p \cdot \mathbf{Q} = 0 \ \textit{where} \ \sum_{s \in S} p_s = 1$$

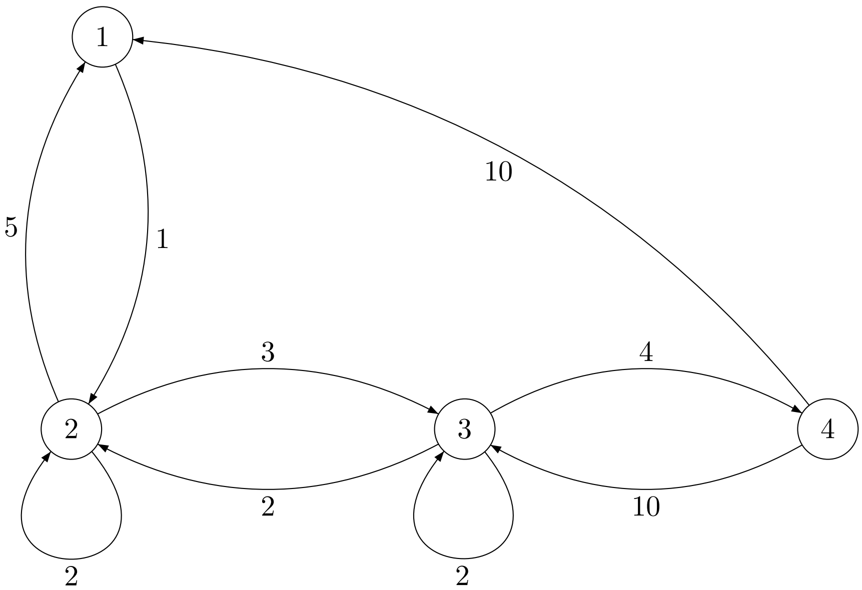
where p is the steady-state probability vector and $\mathbf{Q} = \mathbf{R} - diag(E)$ is the infinitesimal generator matrix.

Exercise 2 **(2 points)**

Prove that exponential distributions are not closed under maximum.

Exercise 3 (2 points)

Uniformize the following CTMC with rate r=20



Exercise 4 (3 points)

A professor supervises three Ph.D. students who all need quite a bit of advice. When any of these students visits the professor, the time to the next visit has an exponential distribution with a mean of 8 hours. The time for the professor to advise the students has a mean value of 1/2 hour. (All times in this problem have exponential distributions). Students visit the professor one at a time. If the professor is busy, the students wait outside his office. And the students are treated in a first-come-first-served manner.

a) Construct the CTMC that describes the situation.

b) At steady state what proportion of the professor's time does he have for himself (without students)?

c) Give the expression of computing the transient distribution of the CTMC at time T. Suppose initially the professor is free. Use the uniformised CTMC.

d) If now comes a new Ph.D. student, who has the same arrival rate as the others, but requires a mean time of one hour with the professor. Do 1 again.