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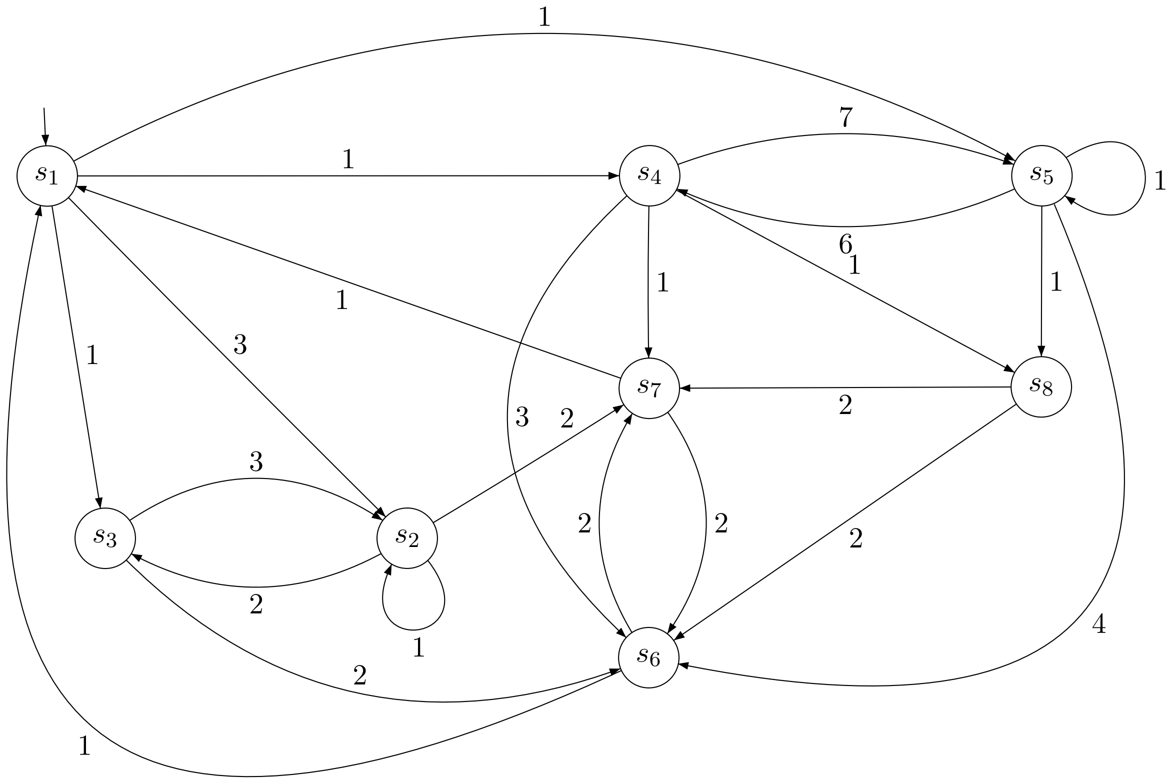
Modeling and Verification of Probabilistic Systems
Summer term 2011

– Series 11 –

Hand in on 6th July before the exercise class.



Exercise 1 **(3 points)**

Consider the CTMC N given in the figure. Let $G = \{s_6, s_7, s_8\}$ be a set of goal states. Find out $sat(\Diamond^{\leq 2} G)$. For this follow the following sets:

- Determine $N/\sim_m$.
- Those equivalent classes that contain goal states, make them absorbing states.
- Uniformize the CTMC obtained in the previous step.
- Find out the transient probability of the uniformized CTMC.

Exercise 2 (3 points)

Let N be a CTMC and R and equivalence relation on S with $(s,t) \in R$. Prove that the following two statements are equivalent:

a) if $\mathbf{P}(s,[s]_R) < 1$ and $\mathbf{P}(t,[t]_R) < 1$, then for all $C \in S/R$, $C \neq [s]_R = [t]_R$:

$\frac{\mathbf{P}(s,C)}{1-\mathbf{P}(s,[s]_R)} = \frac{\mathbf{P}(t,C)}{1-\mathbf{P}(t,[t]_R)}$ and $\mathbf{R}(s, S \backslash [s]_R) = \mathbf{R}(t, S \backslash [t]_R)$

b) $\mathbf{R}(s,C) = \mathbf{R}(t,C)$ for all $C \in S/R$ with $C \neq [s]_R = [t]_R$.

Exercise 3 (2 points)

Let N be a CTMC with state space S, $s, u \in S$, $t \in \mathbb{R}_{\geq 0}$ and let $G \subseteq S$ be closed under $\sim_m$. Prove that:

$s \sim_m u$ implies $Pr(s \models \Diamond^{\leq t} G) = Pr(u \models \Diamond^{\leq t} G)$

Exercise 4 (2 points)

For CTMC N with $s_1, s_2 \in S$: $s_1 \approx_m s_2$ implies $s_1 \approx_p s_2$ in C's embedded DTMC $emb(C)$.