

# Abstraction and Model Checking of CORE ERLANG Programs in MAUDE

– Martin Neuhäuser and Thomas Noll –

Software Modeling and Verification Group (MOVES)  
RWTH Aachen University

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What is CORE ERLANG?

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A strict functional language  
with succinct syntax  
based upon lightweight processes  
and interprocess communication.

## Creation of a new process

The evaluation of the built-in function

```
call 'erlang': 'spawn'(Module, Function_name, Arguments)
```

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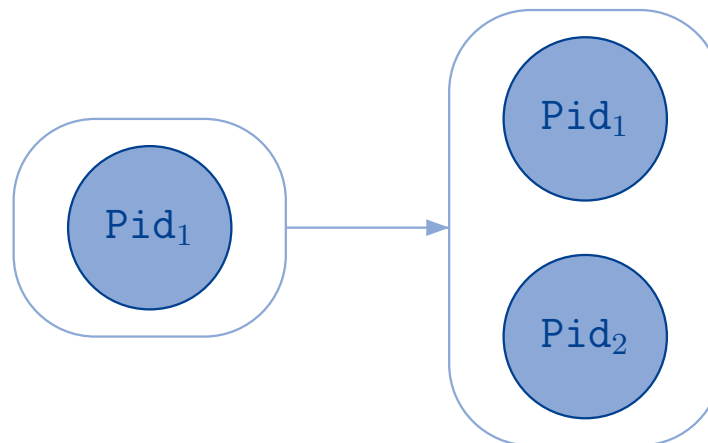
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creates a new process.

- **spawn** returns as soon as the new process is created.
- Evaluates to the **unique identifier** of the created process.
- The new process autonomously starts to evaluate the function call

```
call Module : Function_name (Arguments).
```

- If the evaluation ends, its result is discarded.
  - ↪ Interprocess communication and side effects are a necessity!



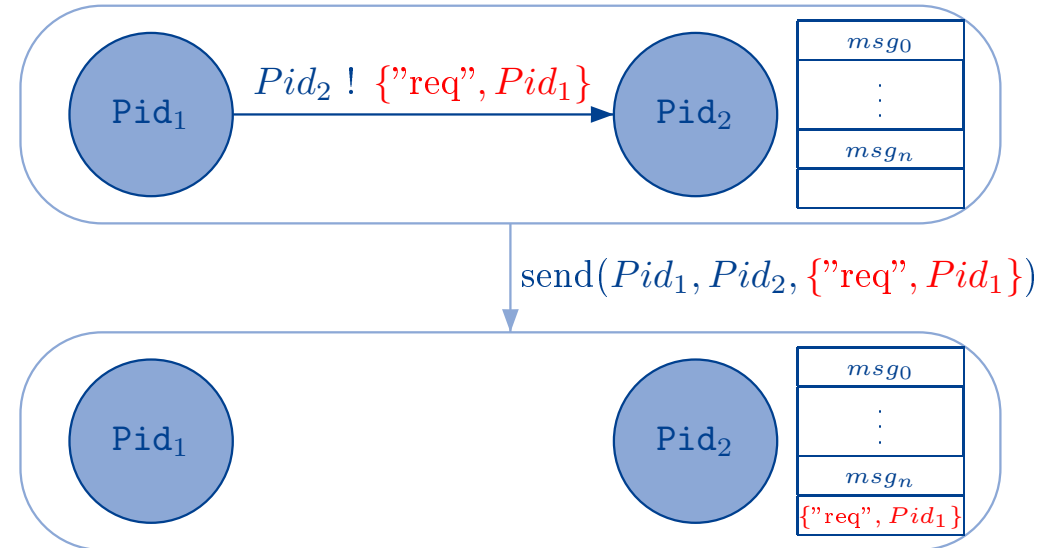
Sending and reception of messages

## • Sending of messages:

The evaluation of an expression

```
call 'erlang': '! ' (Rcv, Expr)
```

- first evaluates its arguments **Rcv** and **Expr**
- and appends the message to the receiver's mailbox.



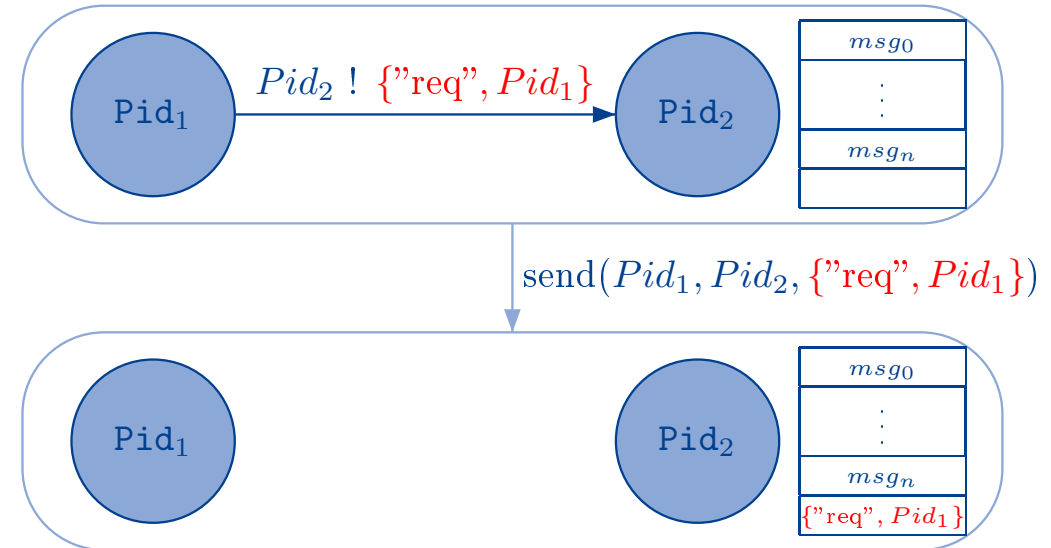
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## • Reception of messages:

**receive**

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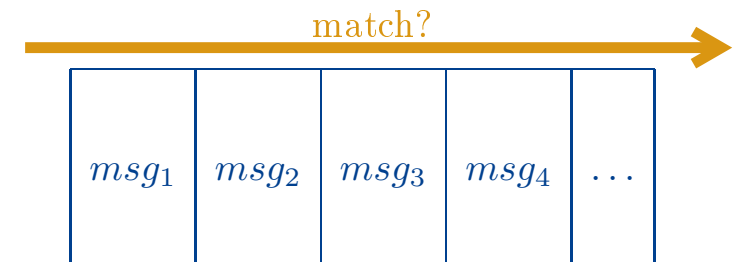
`Pat2 when g2 -> Expr2`

`⋮ ⋮ ⋮`

`Patn when gn -> Exprn`

**after** `Timeout -> TimeoutExpr`

- The oldest matching message is received first.
- Clauses are tried in order of appearance.



## What is it all about?

**Goal:** Verifying properties of Core Erlang programs by means of transition system models

### Approach:

- Formally define the semantics of Core Erlang.
- Operationalize the semantics by transferring it into a Rewriting Logic specification.
- Use abstractions to reduce the state space of the resulting transition systems.
- Automatically derive the transition system model of a given Core Erlang program (MAUDE).

### Verification:

If the set of reachable states is finite, apply model checking techniques to verify properties.



## A first sublanguage: Sequential Core Erlang

- Regard only the **local aspects** of expression evaluation.
- Side effects are formalized by non-determinism.
  - ↪ Non-determinism is resolved later by considering the entire system

Transition system  $T_e$  only captures the local behaviour of an expression!

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## A first example:

- Sequencing operator do:

Example: do 17 apply 'simex'/0()

↪ The first subexpression is fully evaluated. Semantics: Discard its value and continue!

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- **But** what about the evaluation of the **first subexpression**?

**Consider for example:** do call 'erlang':':'(Rcv,Msg) apply 'proceed'/0()

↪ Before evaluation of the do-operator can proceed, its first argument must be evaluated:

$$\frac{e_1 \xrightarrow{e} e'_1}{\underline{\text{do}} e_1 e_2 \xrightarrow{e} \underline{\text{do}} e'_1 e_2} \quad (\text{Seq}_2)$$

## Pattern matching expressions

- case expressions:

$$\frac{\exists i. (\text{match}(val, cl_i) = e' \wedge \forall j < i. \text{match}(val, cl_j) = \perp)}{\text{case } val \text{ of } cl_1 \cdots cl_k \text{ end} \xrightarrow{\tau}_e e'} \quad (\text{Case}_1)$$

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- receive expressions:

qmatch predicate holds iff a matching message is in the mailbox:

$$\text{qmatch}(q, cl_1, \dots, cl_k) := \exists q_1, q_2 \in \text{Const}^*, c \in \text{Const}, i \in \{1, \dots, k\}. \quad q = q_1 \cdot c \cdot q_2 \wedge \text{match}(c, cl_i) \neq \perp$$

Reception of the first matching message ( $c$ ):

$$\frac{\neg \text{qmatch}(q, cl_1, \dots, cl_k) \quad \text{case } c \text{ of } cl_1 \dots cl_k \text{ end} \xrightarrow{\tau}_e e' \quad c_t \in \text{Num} \cup \{\text{'infinity'}\}}{\text{receive } cl_1 \cdots cl_k \text{ after } c_t \rightarrow e_t \xrightarrow{\text{recv}(q, c)}_e e'} \quad (\text{Rcv}_1)$$

**Note:** The prefix  $qc$  of the process' mailbox is guessed nondeterministically!

↪ Reflected by the transition label  $\text{recv}(q, c)$

## Global states and the transition system $T_s$ :

- $\tau$  transitions are autonomous evaluation steps.  
 $\hookrightarrow$  can be lifted to the system layer semantics directly:

$$\frac{e \xrightarrow{\tau}_e e'}{S \cup \{(e, i, q, L, t)\} \xrightarrow{\tau}_s S \cup \{(e', i, q, L, t)\}} \quad (\text{SeqCore})$$

- Sending of messages:  
 $\hookrightarrow$  By considering process systems, we can formalize message transmission:

$$\frac{e_i \xrightarrow{j!c}_e e'_i}{S \cup \{(e_i, i, q_i, L_i, t_i), (e_j, j, q_j, L_j, t_j)\} \xrightarrow{\text{send}(i, j, c)}_s S \cup \{(e'_i, i, q_i, L_i, t_i), (e_j, j, q_j \cdot c, L_j, t_j)\}} \quad (\text{Send}_1)$$

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- Message reception:

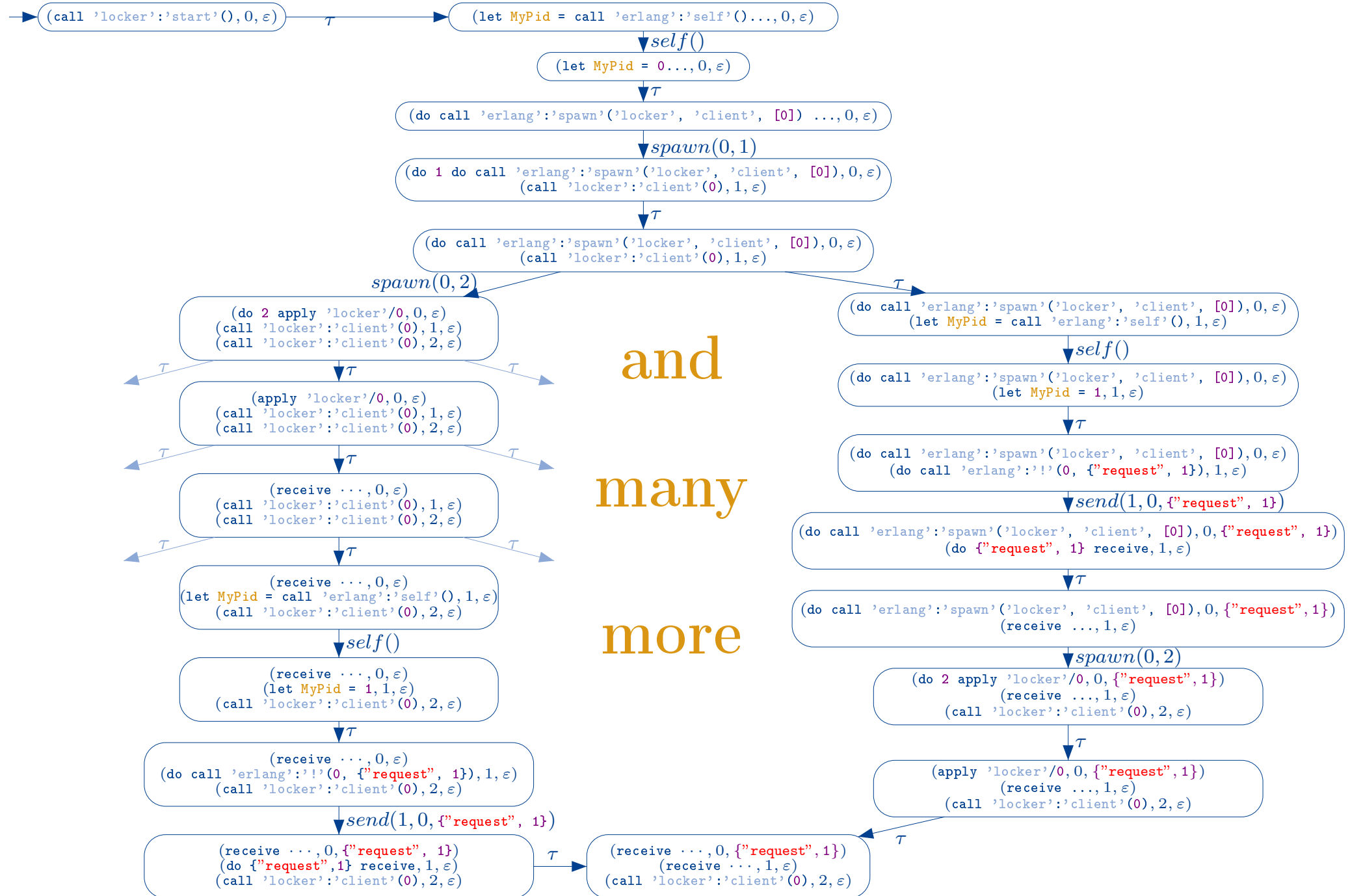
$$\frac{e \xrightarrow{\text{rcv}(q_1, c)}_e e'}{S \cup \{(e, i, q_1 \cdot c \cdot q_2, L, t)\} \xrightarrow{\text{rcv}(i, c)}_s S \cup \{(e', i, q_1 \cdot q_2, L, t)\}} \quad (\text{Rcv})$$

## Example: A simple mutual exclusion protocol in CORE Erlang:

```
'locker'/0 = fun () ->
  receive
    {"request",Client} when 'true' -> do
      call 'erlang':'!'(Client, "ok")
      receive
        {"release",From} when
          call 'erlang':'=='(From,Client)
          -> apply 'locker'/0()
        after 'infinity' -> 'false'
      after 'infinity' -> 'true'
```

```
'client'/1 = fun (LockerPid) ->
  let MyPid = call 'erlang':'self'() in do
    call 'erlang':'!'(LockerPid, {"request", MyPid})
    receive
      "ok" when 'true' -> do
        %% critical section
        call 'erlang':'!'(LockerPid, {"release", MyPid})
        apply 'client'/1(LockerPid)
      after 'infinity' -> 'false'
```



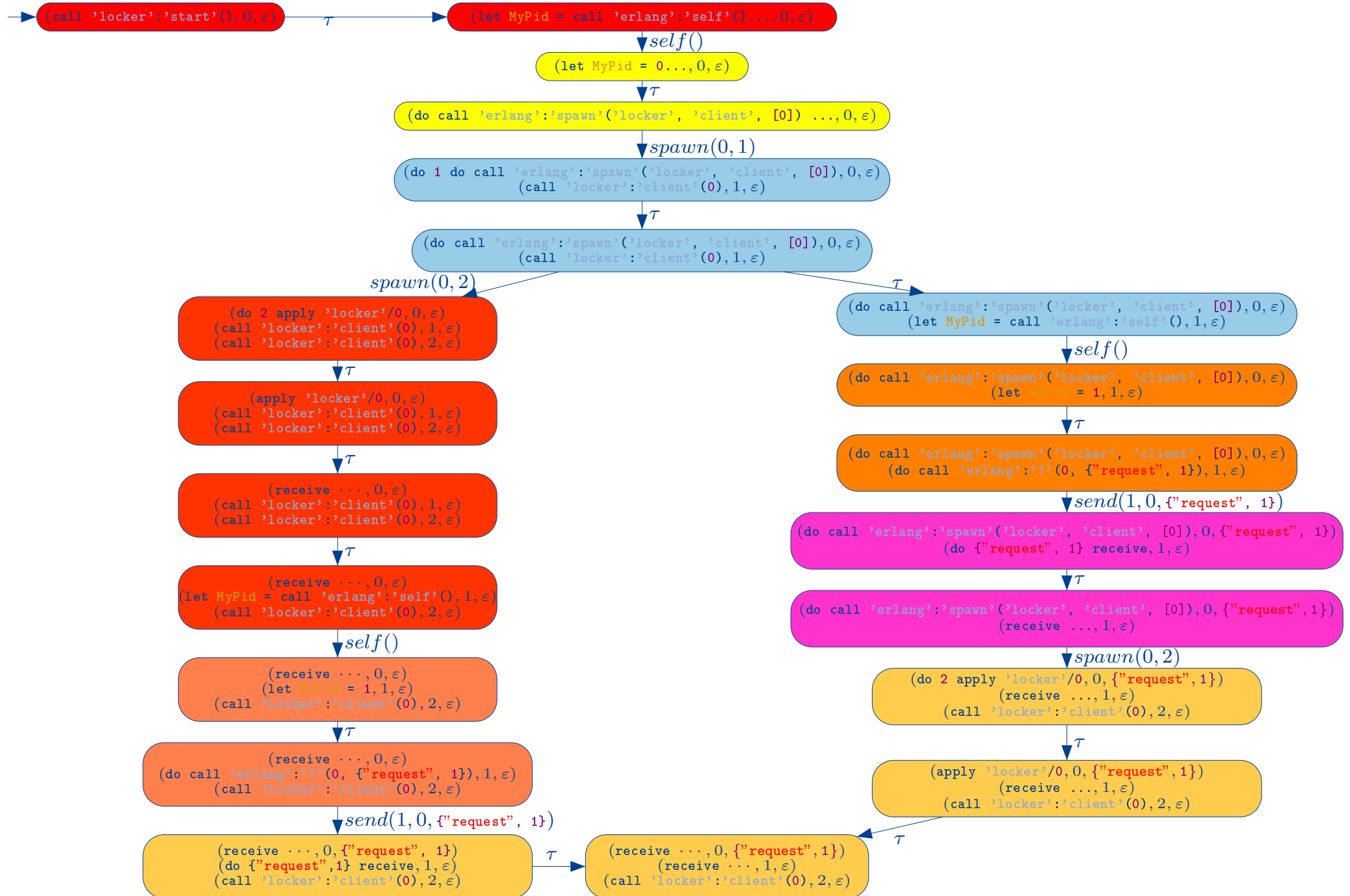


Abstracting from  $\tau$  evaluation steps

$TS_{/\sim} := (\mathcal{S}_{/\sim}, \underline{\text{Act}}, \rightarrow, [s_0]_{\sim})$ , where

- States are the equivalence classes in  $\mathcal{S}_{/\sim}$
- Actions:  $\underline{\text{Act}} := \underline{\text{Act}}_s \setminus \{\tau\}$
- Transition relation  $\rightarrow \subseteq \mathcal{S}_{/\sim} \times \underline{\text{Act}} \times \mathcal{S}_{/\sim}$
- $[s_0]_{\sim} \in \mathcal{S}_{/\sim}$  as initial state

1.  $\overset{\tau}{\longleftrightarrow}_s^*$  denotes the reflexive, symmetric and transitive closure of  $\overset{\tau}{\rightarrow}_s$ .
2. Equivalence relation:  $\sim := \overset{\tau}{\longleftrightarrow}_s^*$



## What is MAUDE?

**Specification language** based on José Meseguer's Rewriting Logic.

**Interpreter** for parameterized Rewriting Logic theories.

Developed at the University of Illinois at Urbana-Champaign.

## MAUDE preliminaries:

1. **Membership equational logic theory**  $(\Omega, \mathcal{E})$  where

- $\Omega = ((\mathcal{K}, \Sigma), \varphi)$  denotes a many kinded signature and
- $\mathcal{E}$  denotes the set of equations.
- $\mathcal{E} = ER \uplus A$  where  $A$  are equational attributes (**associativity, commutativity, identity**) and  $ER$  are (directed) equations  
     $\hookrightarrow$  equational rewriting/simplification

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**Precondition:** The directed equations in  $ER$  are confluent and terminating modulo  $A$

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2. **Rewriting logic theory**  $(\Omega, \mathcal{E}, \phi, R)$  extends the MEL theory:

- $((\mathcal{K}, \Sigma), \varphi)$  is the signature,
- $(\Omega, \mathcal{E})$  is the underlying MEL theory and
- $\phi : \Sigma \rightarrow 2^{\mathcal{N}}$  defines frozen argument positions.
- $R$  denotes the set of rewriting rules  
     $\hookrightarrow$  needs **not** to be confluent!

**Idea:** Normalize term wrt.  $ER \uplus A$  and then apply the rewriting rules from  $R$ !

$\hookrightarrow$  Coherence properties between  $ER \uplus A$  and  $R$  must be fulfilled!

## Representation of processes and process systems in MAUDE

## • Processes:

```
op <□|□|□|□|□|□|□|□> : Label SysResult Expr Pid Mailbox ProcessLinks TrapExit ModEnv -> Process .
```

Label, SysResult and ModEnv are needed in order to operationalize the semantics

## • Process systems:

```
op empty-processes : -> Processes [ctor] .
```

```
op □||□: Processes Processes -> Processes [ctor assoc comm id: empty-processes] .
```

subsort relation:  $\text{Process} \sqsubseteq_{Spec} \text{Processes}$

## • Process environments:

```
op ((□,□,□,□)) : SysLabel Processes ModEnv PidSequence -> ProcessEnvironment .
```

↪ Process environments constitute the states of our transition system.

Specify the equivalence  $\sim$  using the equational theory  $(\Omega, \mathcal{E})$ :

**Example:**

$$\frac{}{\underline{\text{do}} \text{ val } e \xrightarrow{\tau_e} e} \quad (\text{Seq}_1)$$

$$\frac{e_1 \xrightarrow{\alpha_e} e'_1}{\underline{\text{do}} e_1 e_2 \xrightarrow{\alpha_e} \underline{\text{do}} e'_1 e_2} \quad (\text{Seq}_2)$$



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- Evaluation of the do operator itself:

```
eq [norm-do] :
<tau|#no-res|do C EX2|PID|MBOX|LINKS|TRAP|ME> =
<tau|#no-res|EX2|PID|MBOX|LINKS|TRAP|ME> .
```

- Evaluation of the first subexpression:

```
ceq [norm-do] :
<tau|RES|do EX1 EX2|PID|MBOX|LINKS|TRAP|ME> =
<#filterExit(ESL)|RES1|do EX1' EX2|PID|MBOX|LINKS|TRAP|ME>
  if not(EX1 :: Const)
  /\ <ESL|RES1|EX1'|PID|MBOX|LINKS|TRAP|ME> :=
  <tau|RES|EX1|PID|MBOX|LINKS|TRAP|ME> .
```

Rewriting rules define the transition relation  $R_{\rightarrow}$ :

**Idea:** Specify  $\rightarrow \subseteq \mathcal{S}_{/\sim} \times \mathcal{S}_{/\sim}$  by rewriting rules  $R$ !

**Note:** Operationally, process systems are available as normal forms wrt.  $(\Sigma, E \cup A)$  only!

**Example:** Inference rule specifying message reception:

$$\frac{e \xrightarrow{\text{recv}(q_1, c)}_e e'}{S \cup \{(e, i, q_1 \cdot c \cdot q_2, L, t)\} \xrightarrow{\text{recv}(i, c)}_s S \cup \{(e', i, q_1 \cdot q_2, L, t)\}} \quad (\text{Recv})$$

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The corresponding conditional rewrite rule:

```
cr1 [sys-receive] :
  (SL, <receive(C)|#no-res|EX|PID|MBOX|LINKS|TRAP|ME> || PRCS, ME', PIDS) =>
  (sys-receive(PID, C),
    <tau|#no-res|EX|PID|MBOX1|LINKS|TRAP|ME> || PRCS, ME', PIDS)
if MBOX1 := #extractMessage(MBOX|C) .
```

**Remark:** Receivable messages are observed on expression layer but removed on system layer!

## Soundness and completeness

- Semantic point of view:

$$[s]_{\mathcal{E}} = [s]_{A \cup ER} \xrightarrow{R/A \cup ER} [s']_{A \cup ER} = [s']_{\mathcal{E}}$$

- Operational point of view:

$$[s]_A \xrightarrow{ER/A \mid *} \xrightarrow{R/A} [s']_A$$

Do they coincide?

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Do they coincide?

Yes, they do!

## Defining predicates

States of  $TS_{/\sim}$  are represented by  $(\Omega, \mathcal{E})$  normal forms.

$\hookrightarrow$  Associate predicates to these terms:

$s \models \text{send}(i, j, c)$	“process $i$ just sent message $c$ to process $j$ ”
$s \models \text{receive}(i, c)$	“process $i$ just received $c$ ”

**Remark:** If  $s \models \text{send}(i, j, c)$  is valid, the respective state was reached by this transition.

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## Model checking the mutual exclusion protocol:

- As long as the first client is in its critical section, the second cannot enter

$$\varphi_1 = \text{scheduler}(0, 1, 2) \rightarrow \Box (\text{send}(0, 1, \text{"ok"}) \rightarrow (\neg \text{send}(0, 2, \text{"ok"}) \mathcal{U} \text{send}(1, 0, \{\text{"rel"}, 1\})))$$

- Eventually, the second client enters the critical section:

$$\varphi_2 = \text{scheduler}(0, 1) \rightarrow \Diamond (\text{send}(0, 2, \text{"ok"}))$$

$$\varphi_3 = \text{scheduler}(0, 1, 2) \rightarrow \Diamond (\text{send}(0, 2, \text{"ok"}))$$

**But:** In general (unfair scheduling),  $\varphi_2$  is not fulfilled:

$\rightsquigarrow$  Counterexample: The first client enters whereas the second client starves.

## Future Work

1. REAL TIME MAUDE:

Extend the Core Erlang semantics with a notion of time.

2. Case studies:

More examples to see how this approach scales.



Thank you for your attention!

Any questions?

Tool available at <http://www.marneu.com/>