Software-Modellierung und Verifikation
Informatik 2
Prof. J.-P. Katoen
RWTH Aachen

Priv.-Doz. T. Noll noll@cs.rwth-aachen.de
J. Heinen heinen@cs.rwth-aachen.de
Ch. Jansen christina.jansen@cs.rwth-aachen.de

1. Exercise sheet *Static Program Analysis 2011*

Due Mon, 02. May 2011, *before* the exercise course begins.

Exercise 1.1: **(3 points)**

The *available expressions analysis* as presented in the lecture detects for a given program when an expression is available. Modify the analysis in a way that it detects when an expression is available in a *particular variable*: an expression a is available in x at label l if it has been evaluated and assigned to x on all paths leading to l and if the values of x and the variables in the expression have not changed since then.
Develop the dataflow system for this analysis including the kill- and generate-function.

Exercise 1.2: **(3 points)**

Perform a *live variable analysis* for the following program:

$y := 1;$
while $x > 0$ **do** $x := x - 1;$
$y := 2;$

Exercise 1.3: **(1+1+1 points)**

Consider the following labelled program:

$$[x := 1]^1;$$
$$[x := x - 1]^2;$$
$$[x := 2]^3;$$

(a) At which labels is x live, and at which dead? (You need not perform a live variable analysis to confirm your answer!)

(b) Does the result of the live variable analysis always make sense? Point out where the live variable analysis could be improved.

(c) Sketch how the live variable analysis could be adapted/improved with respect to (b) and describe the important changes in more detail.

Exercise 1.4: **(2 points)**

Providing you with a counterexample for the lemma on lecture slide 3.23 (alternative characterisations of of complete lattices):
"Consider the partial order $PO = (D, \subseteq)$ with domain $D = \{\{a\}, \{b\}, \{a, b\}\}$. Obviously, every subset of D has a least upper bound - in 'worst case': $\{a, b\}$. Hence, according to the lemma, PO should be a complete lattice. But the subset $\{\{a\}, \{b\}\} \subset D$ has no greatest lower bound, thus PO is not a complete lattice. Consequently, the lemma is wrong."
Now, find out and explain what is wrong with the lemma given in the lecture or with the counterexample given here.

Exercise 1.5: **(3+1 points)**

Given a partial order $(D, \sqsubseteq)$ a **maximal element** of D is a $d_{\text{max}} \in D$ such that

$$\forall d \in D. d_{\text{max}} \sqsubseteq d \Longrightarrow d_{\text{max}} = d.$$

A **minimal element** of D is a $d_{\text{min}} \in D$ such that

$$\forall d \in D. d \sqsubseteq d_{\text{min}} \Longrightarrow d_{\text{min}} = d.$$

(a) Prove that a partial order $(D, \sqsubseteq)$ with D finite has at least one maximal element.

(b) Let $D = \{\{1,2\}, \{1,2,5\}, \{1,2,3\}, \{2,3\}, \{2,3,5\}\}$ and define $\sqsubseteq$ as the set inclusion. Find all minimal and maximal elements of D.

(c) Can you think of any dependencies between minimal/maximal elements and least upper/greatest lower bounds? Which? Argue why.

Exercise 1.6: **(1+1+1 points)**

Let A be the set of all words in English and
$R = \{(x, y) \in A \times A |$ all the letters in the word x appear, consecutively and in the right order, in the word $y\}$.
Let $B = \{boat, house\}$.

(a) Show that (A, R) is a partial order. Is it a total order? Why?

(b) Does B have any upper or lower bounds?

(c) Is B a complete lattice? Why?

Exercise 1.7: **(1 point)**

Show that the least upper bound of a chain is unique (if it exists).