

6. Exercise sheet *Static Program Analysis 2011*

Due Mon, 6. June 2011, *before* the exercise course begins.

Exercise 6.1:

(3 points)

For a single integer, modulo abstraction is defined by the mapping $\mathbb{Z} \rightarrow \{0, \dots, n-1\} : z \mapsto z \bmod n$ for some fixed $n \geq 1$. Give the definition of the corresponding abstraction and concretization functions operating on sets of integers, and show that they form a Galois connection.

Exercise 6.2:

(2 + 2 points)

Let (α, γ) be a Galois connection with $\alpha : L \rightarrow M$ and $\gamma : M \rightarrow L$.

- (a) Show that $\alpha(\perp_L) = \perp_M$ and $\gamma(\perp_M) = \perp_L$ (where \perp_K is the least element of lattice K).
- (b) Show that $\alpha \circ \gamma \circ \alpha = \alpha$ and $\gamma \circ \alpha \circ \gamma = \gamma$.

Exercise 6.3:

(3 points)

Give a concrete execution of the following WHILE program from any initial state $\sigma \in \Sigma$ with $\sigma(x) = 5$:

```
y := 3;  
z := 0;  
while  x >= y do  
    x := x - y;  
    z := z + 1;
```