

## 6. Exercise sheet *Static Program Analysis 2011*

Due Mon, 6. June 2011, *before* the exercise course begins.

### Exercise 6.1:

(3 points)

For a single integer, modulo abstraction is defined by the mapping  $\mathbb{Z} \rightarrow \{0, \dots, n-1\} : z \mapsto z \bmod n$  for some fixed  $n \geq 1$ . Give the definition of the corresponding abstraction and concretization functions operating on sets of integers, and show that they form a Galois connection.

### Exercise 6.2:

(2 + 2 points)

Let  $(\alpha, \gamma)$  be a Galois connection with  $\alpha : L \rightarrow M$  and  $\gamma : M \rightarrow L$ .

- (a) Show that  $\alpha(\perp_L) = \perp_M$  and  $\gamma(\perp_M) = \perp_L$  (where  $\perp_K$  is the least element of lattice  $K$ ).
- (b) Show that  $\alpha \circ \gamma \circ \alpha = \alpha$  and  $\gamma \circ \alpha \circ \gamma = \gamma$ .

### Exercise 6.3:

(3 points)

Give a concrete execution of the following WHILE program from any initial state  $\sigma \in \Sigma$  with  $\sigma(x) = 5$ :

```

y := 3;
z := 0;
while x >= y do
  x := x - y;
  z := z + 1;

```