Software-Modellierung und Verifikation
Informatik 2
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7. Exercise sheet *Static Program Analysis 2011*

Due Mon, 20. June 2011, *before* the exercise course begins.

Exercise 7.1: **(5 points)**

For every Galois connection we considered so far we observed that $\alpha(\gamma(m)) = m$. This kind of Galois connection is called *Galois insertion*:

(α, γ) is a *Galois insertion* between the complete lattices L and M if and only if:

$$\alpha : L \to M \text{ and } \gamma : M \to L \text{ are monotone functions}$$

that satisfy:

$$\begin{array}{rcll} \gamma(\alpha(l)) & \sqsupseteq & l & \forall\, l \in L \\ \alpha(\gamma(m)) & = & m & \forall\, m \in M \end{array}$$

Show that for a Galois connection (α, γ) between L and M the following claims are equivalent:

(i) (α, γ) is a *Galois insertion*
(ii) γ is injective
(iii) α is surjective
(iv) $\forall\, m_1, m_2 : \; m_1 \sqsubseteq m_2 \Leftrightarrow \gamma(m_1) \sqsubseteq \gamma(m_2)$

Exercise 7.2: **(1 + 2 + 3 points)**

Proof or disproof the following statements:

(a) $\gamma(\top_M) = \top_L$ for any Galois connection (α, γ) with $\alpha : L \to M$ and $\gamma : M \to L$.

(b) Given two Galois connections (α_1, γ_1) and (α_2, γ_2) with $\alpha_1 : L \to M_1$, $\alpha_2 : L \to M_2$, $\gamma_1 : M_1 \to L$ and $\gamma_2 : M_2 \to L$ the *direct product* (α, γ) with:

$$\begin{array}{rcl} \alpha(l) & = & (\alpha_1(l), \alpha_2(l)) \\ \gamma(m_1, m_2) & = & \gamma_1(m_1) \sqcap \gamma_2(m_2)) \end{array}$$

is also a Galois connection.

(c) The *direct product* of two Galois insertions is a Galois insertion.

Exercise 7.3: **(3 + 3 points)**

Consider stacks over integer values. Given a stack s and a number i s.**push(**i**)** pushes i on the top of s, s.**peek()** returns the top most element of s, while s.**pop()** removes the top element without returning it, $t := s$ assigns the stack values from s to the stack t.

(a) Extend the *execution relation* from the lecture for the new operations. Consider val_σ to work on stack variables and **peek()** calls as expected.

(b) Show that there is a reasonable Galois connection between pairs of stacks $L = 2^{\mathbb{Z}^* \times \mathbb{Z}^*}$ and $M = 2^{\{H,S\}} \times 2^{\mathbb{Z}}$, where H means that the top elements of the stacks are equal and S that one stack is a suffix of the other. The second component of M denotes $length(a) - length(b)$, the difference in length for stack pair (a, b). Therefore the value is negative if the first stack contains less elements then the second. Note that the absence of H or S does not claim anything, i.e. neither the absence of H states that the top element is unequal nor the absence o S that none list is a suffix of the other.