

Static Program Analysis

Lecture 6: Dataflow Analysis V

(Constant Propagation & Undecidability of MOP Solution)

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- 1 Repetition: MOP Solution
- 2 Repetition: Constant Propagation
- 3 Example of Constant Propagation Analysis
- 4 Undecidability of the MOP Solution

- Other solution method for dataflow systems
- MOP = Meet Over all Paths
- Analysis information for block B^I = least upper bound over all paths leading to I

Definition (Paths)

Let $S = (L, E, F, (D, \sqsubseteq), \iota, \varphi)$ be a dataflow system. For every $I \in L$, the set of paths up to I is given by

$$\text{Path}(I) := \{[l_1, \dots, l_{k-1}] \mid k \geq 1, l_1 \in E, (l_i, l_{i+1}) \in F \text{ for every } 1 \leq i \leq k, l_k = I\}.$$

For a path $p = [l_1, \dots, l_{k-1}] \in \text{Path}(I)$, we define the transfer function $\varphi_p : D \rightarrow D$ by

$$\varphi_p := \varphi_{l_{k-1}} \circ \dots \circ \varphi_{l_1} \circ \text{id}_D$$

(so that $\varphi_{[]} = \text{id}_D$).

Definition (MOP solution)

Let $S = (L, E, F, (D, \sqsubseteq), \iota, \varphi)$ be a dataflow system where $L = \{I_1, \dots, I_n\}$. The **MOP solution** for S is determined by

$$\text{mop}(S) := (\text{mop}(I_1), \dots, \text{mop}(I_n)) \in D^n$$

where, for every $I \in L$,

$$\text{mop}(I) := \bigsqcup \{\varphi_p(\iota) \mid p \in \text{Path}(I)\}.$$

Remark:

- $\text{Path}(I)$ is generally infinite

⇒ not clear how to compute $\text{mop}(I)$

- In fact: MOP solution generally undecidable (later)

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Formalizing Constant Propagation Analysis I

The **dataflow system** $S = (L, E, F, (D, \sqsubseteq), \iota, \varphi)$ is given by

- set of labels $L := L_c$,
- extremal labels $E := \{\text{init}(c)\}$ (forward problem),
- flow relation $F := \text{flow}(c)$ (forward problem),
- complete lattice (D, \sqsubseteq) where
 - $D := \{\delta \mid \delta : \text{Var}_c \rightarrow \mathbb{Z} \cup \{\perp, \top\}\}$
 - $\delta(x) = z \in \mathbb{Z}$: x has **constant value** z
 - $\delta(x) = \perp$: x **undefined**
 - $\delta(x) = \top$: x **overdefined** (i.e., different possible values)
 - $\sqsubseteq \subseteq D \times D$ defined by pointwise extension of $\perp \sqsubseteq z \sqsubseteq \top$
(for every $z \in \mathbb{Z}$)

Example

$$\begin{aligned}\text{Var}_c &= \{\mathbf{w}, \mathbf{x}, \mathbf{y}, \mathbf{z}\}, \\ \delta_1 &= (\underbrace{\perp}_{\mathbf{w}}, \underbrace{1}_{\mathbf{x}}, \underbrace{2}_{\mathbf{y}}, \underbrace{\top}_{\mathbf{z}}), \quad \delta_2 = (\underbrace{3}_{\mathbf{w}}, \underbrace{1}_{\mathbf{x}}, \underbrace{4}_{\mathbf{y}}, \underbrace{\top}_{\mathbf{z}}) \\ \implies \delta_1 \sqcup \delta_2 &= (\underbrace{3}_{\mathbf{w}}, \underbrace{1}_{\mathbf{x}}, \underbrace{\top}_{\mathbf{y}}, \underbrace{\top}_{\mathbf{z}})\end{aligned}$$

Dataflow system $S = (L, E, F, (D, \sqsubseteq), \iota, \varphi)$ (continued):

- extremal value $\iota := \delta_{\top} \in D$ where $\delta_{\top}(x) := \top$ for every $x \in \text{Var}_c$
(i.e., every x has (unknown) default value)
- transfer functions $\{\varphi_I \mid I \in L\}$ defined by

$$\varphi_I(\delta) := \begin{cases} \delta & \text{if } B^I = \text{skip} \text{ or } B^I \in BExp \\ \delta[x \mapsto \text{val}_{\delta}(a)] & \text{if } B^I = (x := a) \end{cases}$$

where

$$\begin{aligned} \text{val}_{\delta}(x) &:= \delta(x) & \text{val}_{\delta}(a_1 \text{ op } a_2) &:= \begin{cases} z_1 \text{ op } z_2 & \text{if } z_1, z_2 \in \mathbb{Z} \\ \perp & \text{if } z_1 = \perp \text{ or } z_2 = \perp \\ \top & \text{otherwise} \end{cases} \\ \text{val}_{\delta}(z) &:= z \end{aligned}$$

for $z_1 := \text{val}_{\delta}(a_1)$ and $z_2 := \text{val}_{\delta}(a_2)$

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Example of Constant Propagation Analysis

Example 6.1

Constant Propagation Analysis for

$c := [x := 1]^1;$	$\varphi_1((a, b, c, d)) = (a, 1, c, d)$
$[y := 1]^2;$	$\varphi_2((a, b, c, d)) = (a, b, 1, d)$
$[z := 1]^3;$	$\varphi_3((a, b, c, d)) = (a, b, c, 1)$
$\text{while } [z > 0]^4 \text{ do}$	$\varphi_4((a, b, c, d)) = (a, b, c, d)$
$[w := x+y]^5;$	$\varphi_5((a, b, c, d)) = (b + c, b, c, d)$
$\text{if } [w = 2]^6 \text{ then}$	$\varphi_6((a, b, c, d)) = (a, b, c, d)$
$[x := y+2]^7$	$\varphi_7((a, b, c, d)) = (a, c + 2, c, d)$

- ➊ Fixpoint solution (on the board)
- ➋ MOP solution (on the board)

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Theorem 6.2 (Undecidability of MOP solution)

The MOP solution for Constant Propagation is undecidable.

Proof.

Based on undecidability of **Modified Post Correspondence Problem**:

Let Γ be some alphabet, $n \in \mathbb{N}$, and $u_1, \dots, u_n, v_1, \dots, v_n \in \Gamma^+$.

Do there exist $i_1, \dots, i_m \in \{1, \dots, n\}$ with $m \geq 1$ and $i_1 = 1$ such that

$u_{i_1} u_{i_2} \dots u_{i_m} = v_{i_1} v_{i_2} \dots v_{i_m}$?

(on the board)

