

## 2. Exercise sheet *Semantics and Verification of Software 2007*

Due to Wed., 25 April 2007, *before* the exercise course begins.

### Exercise 2.1:

Show that the bigstep relation and the singlestep relation on arithmetic expressions, as defined in Exercise 1.2, are equivalent, i.e., that for every  $a \in AExp$ ,  $\delta \in \Sigma$ , and  $z \in \mathbb{Z}$ :

$$\langle a, \delta \rangle \rightarrow z \quad \text{iff} \quad \langle a, \delta \rangle \rightarrow_1^* z.$$

### Exercise 2.2:

Construct the derivation tree for the operational semantics of the WHILE statement

$$z := 0; \textbf{while } y \leq x \textbf{ do } (z := z + 1; x := x - y)$$

starting in a state  $\delta \in \Sigma$  with  $\delta(x) = 12$  and  $\delta(y) = 5$ .

### Exercise 2.3:

Show that the statement  $c \in Cmd$  given by

$$y := 1; \textbf{while } \neg(x = 1) \textbf{ do } (y := y * x; x := x - 1)$$

computes the factorial, i.e., that its operational semantics satisfies the following condition:

$$\mathfrak{D}[\![c]\!](\delta)(y) = (\delta(x))!$$

for every  $\delta \in \Sigma$  with  $\delta(x) \geq 1$ .

### Exercise 2.4:

- (a) Extend the WHILE language by a loop construct of the form

$$\textbf{repeat } c \textbf{ until } b$$

and define its execution relation  $\rightarrow$  without (explicitly) using the **while** statement.

- (b) Establish the following semantic equivalence:

$$\textbf{repeat } c \textbf{ until } b \quad \sim \quad c; \textbf{ if } b \textbf{ then skip else } (\textbf{repeat } c \textbf{ until } b).$$

- (c) Establish the following semantic equivalence:

$$\textbf{repeat } c \textbf{ until } b \quad \sim \quad c; \textbf{ while } \neg b \textbf{ do } c.$$