

2. Exercise sheet *Semantics and Verification of Software 2007*

Due to Wed., 25 April 2007, *before* the exercise course begins.

Exercise 2.1:

Show that the bigstep relation and the singlestep relation on arithmetic expressions, as defined in Exercise 1.2, are equivalent, i.e., that for every $a \in AExp$, $\delta \in \Sigma$, and $z \in \mathbb{Z}$:

$$\langle a, \delta \rangle \rightarrow z \quad \text{iff} \quad \langle a, \delta \rangle \rightarrow_1^* z.$$

Exercise 2.2:

Construct the derivation tree for the operational semantics of the WHILE statement

$$z := 0; \mathbf{while} \ y \leq x \ \mathbf{do} \ (z := z + 1; \ x := x - y)$$

starting in a state $\delta \in \Sigma$ with $\delta(x) = 12$ and $\delta(y) = 5$.

Exercise 2.3:

Show that the statement $c \in Cmd$ given by

$$y := 1; \mathbf{while} \ \neg(x = 1) \ \mathbf{do} \ (y := y * x; \ x := x - 1)$$

computes the factorial, i.e., that its operational semantics satisfies the following condition:

$$\mathfrak{O}[c](\delta)(y) = (\delta(x))!$$

for every $\delta \in \Sigma$ with $\delta(x) \geq 1$.

Exercise 2.4:

(a) Extend the WHILE language by a loop construct of the form

repeat c **until** b

and define its execution relation \rightarrow without (explicitly) using the **while** statement.

(b) Establish the following semantic equivalence:

repeat c **until** b \sim $c; \mathbf{if} \ b \ \mathbf{then} \ \mathbf{skip} \ \mathbf{else} \ (\mathbf{repeat} \ c \ \mathbf{until} \ b).$

(c) Establish the following semantic equivalence:

repeat c **until** b \sim $c; \mathbf{while} \ \neg b \ \mathbf{do} \ c.$