Semantics and Verification of Software

Lecture 13: Semantics of Blocks and Procedures

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Winter semester 2008/09

Outline

Blocks and Procedures

- Extension of WHILE by blocks with (local) variables and (recursive) procedures

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- Involves new semantic concepts:
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  - locations (memory addresses) and stores (memory states)
- Important: scope of variable and procedure identifiers

  static scoping: scope of identifier = declaration environment (here)
  dynamic scoping: scope of identifier = calling environment
  (old Algol/Lisp dialects)

Static and Dynamic Scoping

Example 13.1

```
begin
  var x; var y;
  proc P is y := x;
  x := 1;
  begin
    var x;
    x := 2;
    call P
  end
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dynamic scoping $\Longrightarrow$ y = 2

Extending the Syntax

Syntactic categories:

Category	Domain	Meta variable
Procedure identifiers	$PVar = \{\mathtt{P}, \mathtt{Q}, \ldots\}$	P
Procedure declarations	$PDec$	p
Variable declarations	$VDec$	v
Commands (statements)	Cmd	c

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Context-free grammar:

$$p ::= \texttt{proc}\ P\ \texttt{is}\ c\texttt{;}p \mid \varepsilon \in PDec$$
$$v ::= \texttt{var}\ x\texttt{;}v \mid \varepsilon \in VDec$$
$$c ::= \texttt{skip} \mid x\ \texttt{:=}\ a \mid c_1\texttt{;}c_2 \mid \texttt{if}\ b\ \texttt{then}\ c_1\ \texttt{else}\ c_2 \mid \texttt{while}\ b\ \texttt{do}\ c \mid \texttt{call}\ P \mid \texttt{begin}\ v\ p\ c\ \texttt{end} \in Cmd$$

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- Thus: previous state information represented as $\sigma \circ \rho$

Operational Semantics II

- Effect of procedure call determined by its body statement and variable and procedure environment of its declaration:

$$PEnv := \{\pi \mid \pi : PVar \dashrightarrow Cmd \times VEnv \times PEnv\}$$

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- Effect of declaration: update of environment

$$\begin{aligned}
\mathsf{upd}_v[\![.]\!] &: VDec \times VEnv \times Sto \to VEnv \times Sto \\
\mathsf{upd}_v[\![\texttt{var } x\texttt{;}v]\!](\rho,\sigma) &:= \mathsf{upd}_v[\![v]\!](\rho[x \mapsto l_x], \sigma[l_x \mapsto 0]) \\
\mathsf{upd}_v[\![\varepsilon]\!](\rho,\sigma) &:= (\rho,\sigma) \\
\mathsf{upd}_p[\![.]\!] &: PDec \times VEnv \times PEnv \to PEnv \\
\mathsf{upd}_p[\![\texttt{proc } P \texttt{ is } c\texttt{;}p]\!](\rho,\pi) &:= \mathsf{upd}_p[\![p]\!](\rho, \pi[P \mapsto (c,\rho,\pi)]) \\
\mathsf{upd}_p[\![\varepsilon]\!](\rho,\pi) &:= \pi
\end{aligned}$$

  where $l_x := \min\{l \in \mathbb{N} \mid \sigma(l) = \bot\}$

Execution Relation I

Definition 13.2 (Execution relation)

For $c \in Cmd$, $\sigma, \sigma' \in Sto$, $\rho \in VEnv$, and $\pi \in PEnv$, the execution relation $(\rho, \pi) \vdash \langle c, \sigma \rangle \to \sigma'$ is defined by the following rules:

$$\text{(skip)}\ \frac{}{(\rho, \pi) \vdash \langle \texttt{skip}, \sigma \rangle \to \sigma}$$

$$\text{(asgn)}\ \frac{\langle a, \sigma \circ \rho \rangle \to z}{(\rho, \pi) \vdash \langle x\ \texttt{:=}\ a, \sigma \rangle \to \sigma[\rho(x) \mapsto z]}$$

$$\text{(seq)}\ \frac{(\rho, \pi) \vdash \langle c_1, \sigma \rangle \to \sigma' \quad (\rho, \pi) \vdash \langle c_2, \sigma' \rangle \to \sigma''}{(\rho, \pi) \vdash \langle c_1\,\texttt{;}\,c_2, \sigma \rangle \to \sigma''}$$

$$\text{(if-t)}\ \frac{\langle b, \sigma \circ \rho \rangle \to \mathsf{true} \quad (\rho, \pi) \vdash \langle c_1, \sigma \rangle \to \sigma'}{(\rho, \pi) \vdash \langle \texttt{if}\ b\ \texttt{then}\ c_1\ \texttt{else}\ c_2, \sigma \rangle \to \sigma'}$$

$$\text{(if-f)}\ \frac{\langle b, \sigma \circ \rho \rangle \to \mathsf{false} \quad (\rho, \pi) \vdash \langle c_2, \sigma \rangle \to \sigma'}{(\rho, \pi) \vdash \langle \texttt{if}\ b\ \texttt{then}\ c_1\ \texttt{else}\ c_2, \sigma \rangle \to \sigma'}$$

Execution Relation II

Definition 13.2 (Execution relation; continued)

$$\text{(wh-f)}\ \frac{\langle b, \sigma \circ \rho\rangle \to \mathsf{false}}{(\rho,\pi) \vdash \langle \texttt{while}\ b\ \texttt{do}\ c, \sigma\rangle \to \sigma}$$

$$\text{(wh-t)}\ \frac{\langle b, \sigma \circ \rho\rangle \to \mathsf{true}\ (\rho,\pi) \vdash \langle c, \sigma\rangle \to \sigma'\ (\rho,\pi) \vdash \langle \texttt{while}\ b\ \texttt{do}\ c, \sigma'\rangle \to \sigma''}{(\rho,\pi) \vdash \langle \texttt{while}\ b\ \texttt{do}\ c, \sigma\rangle \to \sigma''}$$

$$\text{(call)}\ \frac{(\rho', \pi'[P \mapsto (c, \rho', \pi')]) \vdash \langle c, \sigma\rangle \to \sigma'}{(\rho,\pi) \vdash \langle \texttt{call}\ P, \sigma\rangle \to \sigma'} \qquad \text{if } \pi(P) = (c, \rho', \pi')$$

$$\text{(block)}\ \frac{\mathsf{upd}_v[\![v]\!](\rho,\sigma) = (\rho',\sigma') \quad (\rho', \mathsf{upd}_p[\![p]\!](\rho',\pi)) \vdash \langle c, \sigma'\rangle \to \sigma''}{(\rho,\pi) \vdash \langle \texttt{begin}\ v\ p\ c\ \texttt{end}, \sigma\rangle \to \sigma''}$$

Execution Relation III

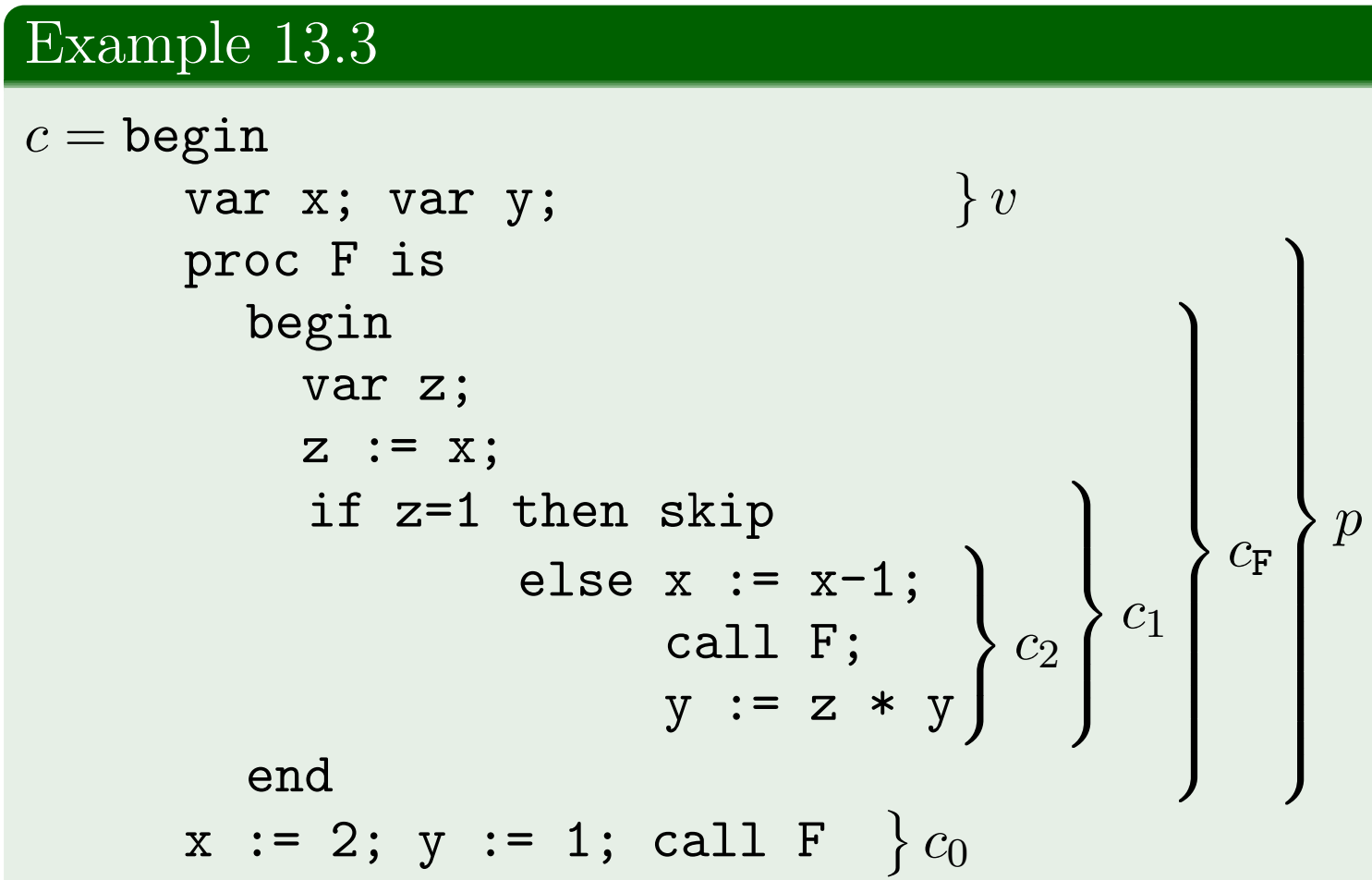
Remarks about rules (call) and (block):

- Static scoping is modelled in (call) by using the environments ρ' and π' (as determined in (block)) from the declaration site of procedure P (and not ρ and π from the calling site)
- In (call), the procedure environment associated with procedure P is extended by a P-entry to handle recursive calls of P:

$$\pi'[P \mapsto (c, \rho', \pi')]$$

Execution Relation IV

Example 13.3



Let $\sigma_\emptyset(l) = \rho_\emptyset(x) = \pi_\emptyset(P) = \perp$ for all $l \in Loc, x \in Var, P \in PVar$

Notation: $\sigma_{ijkl} \Leftrightarrow \sigma(0) = i, \sigma(1) = j, \sigma(2) = k, \sigma(3) = l$

Derivation tree for $(\rho_\emptyset, \pi_\emptyset) \vdash \langle c, \sigma_\emptyset \rangle \to \sigma_{1221}$: on the board